A MODEL FOR THE CURRENT TRANSPORT IN HIGH-TEMPERATURE SUPERCONDUCTING TAPES

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A new model is developed to describe the low-temperature transport properties of polycrystalline HTSC ceramics at high magnetic fields. It is concluded that if the Abrikosov vortices in the grains are rigidly pinned, the double-step character of $J_c(H)$ vs H dependence and abnormal hysteresis can be observed. Evidence for surface barriers to vortex motion in the grains of YBCO-123 metal ceramics is summarized. Characteristic features of this barrier appear in the abnormal hysteresis loop of the critical transport current.

1. Introduction

Grain-oriented films have shown convincingly the ability of metal ceramic materials to conduct considerable current without dissipation [1]. The difficulties associated with flux-line lattice instability in ideal single crystals of metal ceramics have been successfully avoided by the introduction of additional pinning centers (for example, by neutron irradiation of the crystals [2]). But the situation for the long-length tapes is much more complicated. The contacts between granules in these materials must have good electrical properties. In Y- and Bi-metal ceramics this is prevented by the extremely small coherence length ξ and the structure peculiarity (Zandbergen et al. [3]). At such distances, particularly in the region of structure defects, stoichiometry composition of ceramics, its oxygen content can alter, which is reflected in the local values of the critical temperature and order parameter. As a result, the superconducting metal oxide characteristics prove to be sensitive to all kinds of atomic defects — dislocations, small-angle boundaries and other packing defects, twinning planes and all the more — to such important structure disturbances that are the weak-linked boundaries. The situation becomes complicated owing to the sensitivity of ceramic contact properties to the matching of directions of the crystallographic axes of granules (Dimos et al. [4]).

By orientating the grains one can obtain samples with well co-orientating (in line with one another) granules ("the alignment" phenomenon [5]). In our opinion, with this technology, however, there are no prospects of fulfilling the main standards for long-length wire samples, i.e. the high efficiency of the technological process and the ability of the products to withstand considerable bending strains.

The fulfillment of these requirements with the advantages of the "alignment" of the grain (as in the Bi-based tapes, Heine et al. [6]) is complicated through loosing contact of technology researches with understanding of the Josephson medium physics. This complicates the choice of perspective trends through which the achievement of success is the most probable.

For example, now one fails to attain satisfactory understanding of the nature of the current state in metal oxide samples in high magnetic fields. Usually it is assumed that the magnetic field H considerably influences the critical current J_c of ceramics superconductor at fields $H \geq H_0 \cong \Phi_0/(2\lambda W)$, where Φ_0 is the magnetic flux quantum, W is the contact width and λ is the London penetration depth into the granules. For metal ceramics the "critical" field H_0 is too small, $H_0 \sim 10$ Oe [7].

The strong $J_c - H$ dependence weakens for $H > H_0$ if there are spatially inhomogeneous of the Josephson contacts critical current density. In that case the H_0 value is reduced to $H_0 \cong \Phi_0/(2\lambda r)$ where r is the correlation radius of the spatial fluctuations of the Josephson critical current (or the small contacts width) [8–10]. But there is good reason to believe that the main contribution to the Josephson medium critical current stabilization is given by the penetration of Abrikosov vortices into the granules. This problem was considered early [11–13] but without taking into consideration the surface barriers effects. The surface barrier to vortex motion has recently come into evidence in high-temperature superconductors (Malozemoff, [14]).

In this paper a new model is developed to describe the field dependence of critical current of a Josephson media in high magnetic fields $H >> H_{c1}$, which takes into account the surface barrier effects. A good agreement between the proposed theory and experiments [15,16] has been achieved.

2. Dependence of critical current on parallel magnetic field

It is conventionally assumed that the weak links are the origin of the low critical current density J_c HTSC-ceramics at high magnetic fields. Really, magnetic field considerably influences the critical current I_c of Josephson junctions at fields $\mu_0 H \sim \Phi_0/S_0$. Therefore the often observed double-step behaviour and plateau region in the $J_c(B)$ curves are interpreted as a result of the percolation paths that remain after the Josephson decoupling in high magnetic fields has occurred [7,15].

It is known that field H essentially suppresses the Josephson current if full phase difference $\Delta\varphi$ along the contacts areas is of the order of π, i.e. $\Delta\varphi \cong W(d\varphi/dx) \approx \pi$. If there are no Abrikosov vortices in granules, then $d\varphi/dx = (4\pi\lambda^2\mu_0/\Phi_0)H/\lambda$, so the conventional ratio $B_0 \approx \Phi_0/(2\lambda W)$ takes place. In presence of the intragranular vortices, a correct estimate of the modification of the Ferrell–Prange equations must be done.

There is the total connections between the gradient-invariant vector potential $\mathrm{Q} = \nabla\chi - (2\pi/\Phi_0)\mathrm{A}$ (the full phase difference is $\Delta\varphi = \int \mathrm{Q}dl$) and the screening current $j_s \equiv j_{sf} \sim Q$ on the surface of contact electrodes:

$$\oint_C \mathrm{Q}dl = \int_S \mathrm{rotQ}ds = -\left[(2\pi/\Phi_0)Bd_N\right]dx = \varphi(x+dx) - \varphi(x) - \left[4\pi\lambda^2\mu_0/\Phi_0\right]j_{sf}dx\,,$$

where the contour C passes on the surface of the electrodes between the point x and $x + dx$, so the area s is $s = dx \cdot t$, $\mathrm{B} = \mu_0\mathrm{H} = \mathrm{rot\,A}$, and the London equation $j_{sf} = (PHI_0/2\pi\mu_0\lambda^2)\mathrm{Q}$ is used. Thus, the full change of the phase difference $\varphi(x)$ across the junction is [9]

$$\Delta\varphi \equiv Wd\varphi/dx = \frac{2\pi\mu_0}{\Phi_0}W(2\lambda^2 j_{sf} + d_N H) \tag{1}$$

where the direction X is perpendicular to the external field $H = H_z$. If there are no Abrikosov vortices in the granules, then j_s value is given by the external field H_e,

$j_s = dH/dy = H_e/\lambda$ and $\Delta\varphi \sim (2\lambda + t)H_e$. After the Abrikosov vortices have entered the granules, the j_s value is no longer proportional to the external field H_e, and the phase difference $\Delta\varphi$ (1) depends now on the surface critical current amplitude j_{sf} of the grains.

In a general case the surface screening current j_{sf} has the Meissner part of surface current j_{sm} and the irreversible one, j_{cg}, determined by the magnetic flux pinning in the vicinity of the granule surface. The sign of j_{cg} depends on the magnetic history of the sample.

If we use the boundary condition that at the granule surface ($x = 0$) the value of induction B(0) and the external field H_e are in equilibrium [17]

$$H_e \cong B(0)/\mu_0 + M_{eq}, \tag{2}$$

where M_{eq} is the equilibrium (Abrikosov) diamagnetic magnetization, then [8,11]

$$j_{sf}(H) \cong j_{sm} \pm j_{cg} \cong M_{eq}/\lambda \pm j_{cg}. \tag{3}$$

For $H > H_{c1}$ the M_{eq} value is almost constant vs. field H_e,

$$M_{eq}(H_e) \approx \Phi_0/(8\pi\lambda^2\mu_0)\ln(H_{c2}/H_e)(1 - H_e/H_{c2}) \cong H_{c1}$$

and $j_{sm} \approx$ constant (H_e). As a result, for fields region $H_{c1} << H_e \le H_w = \Phi_0/(\mu_0 \cdot Wd_N)$ the $\Delta\varphi$ value (1) is approximately constant vs. H_e and the Josephson medium critical current J_c vs. H dependence has a double-step character [11–13]. The first drop in J_c is for $H \ge H_0 = \Phi_0/\{\mu_0(2\lambda + d_N)W\}$. This is followed by a plateau regime at magnetic fields $H_p \le H \le H_{j2}$, $H_p = Lj_{cg}$, where

$$H_{j2} = \Phi_0/(\mu_0 2\pi r d_N) + (2\lambda^2/d_N)\, j_{cg} + (2\lambda/d_N)H_{c1}. \tag{4}$$

The second drop in J_c occurs at high enough fields $H \ge H_{j2}$. Here r is the correlation radius of the spatial fluctuations of the Josephson critical current and we have assumed that for $H >> H_{c1}$ the intergrain field is roughly equal to H_e. If the Josephson junction effective width r and thickness d_N are small, $r << \lambda$ and $d_N << \lambda$, then $H_{j2} >> H_{c1}$.

For example, let magnetic field H be parallel to the ab plane of YBCO granule and the Josephson junction exists in the c-direction. Then $\lambda \cong \lambda_a$, $H_{c1} \equiv H_{c1a} \approx \Phi_0 \ln(\kappa)/(4\pi\mu_0\lambda_a\lambda_c)$, and for $\lambda_a = 0.15\,\mu m$, $\Gamma = \lambda_c/\lambda_a = 5$, $\xi_a = 1.5\,nm$, $d_N = 2\,nm$, $r = 0.1\,\mu m$, critical current value in the ab-plane $j_{cg} = 2 \cdot 10^5$ A, the $B_{j2} \equiv \mu_0 H_{j2}$ value is equal to $B_{j2} \cong 2.7$ T.

If there exists a surface barrier to Abrikosov vortex motion the equilibrium part of the Meissner path of surface current j_{sm} for field increasing and field decreasing processes is different. As a result, a new mechanism of the Josephson critical current hysteresis appears. In the presence of a barrier against flux entry or exit the boundary condition on $H_e(0)$ (2), where flux has entered is now

$H_e = H_{en}[B_\uparrow(0)]$, and the condition where flux has exited is $H_e = H_{ex}[B_\downarrow(0)]$ [18].

As distinct from the equilibrium case, when the Meissner current value is fixed by the equilibrium magnetization M_{eq}, in the presence of a surface barrier the j_{sm} value depends on the distance x_0 of the first Abrikosov vortices layers on the surface

$$j_{sm} = [H - (B/\mu_o) \exp\{-x_v/\lambda_a\} x_\lambda/\sinh(x_\lambda)] . \tag{5}$$

Here $x_v = x_0 - h_a/2$, $x_\lambda = h_a/2\lambda_a$, h_a is the distance between Abrikosov vortex in the direction normal to the granule surface,

$$h_a \approx \left[\Phi_0/B\Gamma 2\sqrt{3}\right]^{1/2}$$

(Kogan [19], Ivlev and Kopnin [20]). The x_v value is found from the Gibbs free energy minimization

$$x_v = \lambda_a \cdot \cosh^{-1}[(\mu_0 H_e/B)\{\sinh(x_\lambda)/x_\lambda\}] , \tag{6}$$

and (for field-increasing process) we receive the surface current

$$j_{sf}^{\downarrow} \cong M_{en}/\lambda_a, \quad M_{en}(B) = \left\{H_{en}^2 - \left[\frac{B x_\lambda}{\mu_0 \sinh(x_\lambda)}\right]^2\right\}^{1/2} , \tag{7}$$

and for field-decreasing process it is

$$j_{sf}^{\uparrow} \cong M_{ex}/\lambda_a, \quad M_{ex}(B) = \left\{H_{ex}^2 - \left[\frac{B x_\lambda}{\mu_0 \sinh(x_\lambda)}\right]^2\right\}^{1/2} . \tag{8}$$

(Clem used the continual model for the vortex density distribution, so in formulas (6–8) the term $x_\lambda/\sinh(x_\lambda)$ was absent [21].)

In presence of the surface barrier the vortex contribution to the surface current is given by $\pm j_{cg} \cdot \exp(-x_v/\lambda)$, so the full surface current value is

$$j_{sf}^{\downarrow\uparrow} = M^{(en)(ex)}/\lambda_a \pm j_{cg} \cdot \exp(-x_v/\lambda_a) . \tag{9}$$

Here for field increasing case we have used the generalized square-root model [21]

$$H_{en}(B^\uparrow) = \left[H_s^2 + k_{en}(B/\mu_0)^2\{x_\lambda/\sinh(x_\lambda)\}^2\right]^{1/2}, \quad k_{en} = 1 - (H_s/H_{c2})^2, \tag{10}$$

$H_s = \Phi_0/(4\pi\mu_0\lambda_a\xi_a)$, H_{c2} is the upper critical field for the grains, and for decreasing external field

$$H_{ex}(B_\downarrow) = B/\mu_0 + H_{c1} . \tag{11}$$

For spatially inhomogeneous contacts the Yanson relation is valid [9]

$$I_c(H) \cong \frac{I_{sf}}{\left[1+\Delta\varphi^2\right]^{1/2}} = \frac{I_{cf}}{\left[1+(H/H_{c2}^{cer})^2\right]^{1/2}} . \tag{12}$$

Where $H_{c2}^{cer} = \Phi_0/(4\pi\mu_0\lambda r)$. After the Abrikosov vortices have entered granules,

$$\Delta\varphi = r\left[2\pi\mu_0/\Phi_0\{2\lambda^2 j_{sf} + d_N H\}\right], \quad (13)$$

where the surface current value j_{sf} is given by the formula (3) for equilibrium boundary conditions (2) and by the formula (9) if there is the surface barrier.

According to (9)–(10) in increasing external fields the $\Delta\varphi$ value (13) is approximately constant vs. the external field H_e for the region $H_s \le H_e \le H_{j2}$, where

$$H_{j2} = \Phi_0/\left[\mu_0 2\pi r d_N\right] + (2\lambda^2/d_N)j_{cg} + (2\lambda/d_N)H_s. \quad (14)$$

As before, the Josephson medium critical current J_c vs. H dependence has a double-step character (compare (4) with (14)), but in the presence of a barrier against flux entry into granules for increasing fields the plateau region $H_s \le H_e \le H_{j2}$ is considerably large when

$$H_s = \Phi_0/(4\pi\mu_0\lambda_a\xi_a) >> H_{c1} = \Phi_0 \ln(\kappa)/(4\pi\mu_0\lambda_a\lambda_c),$$

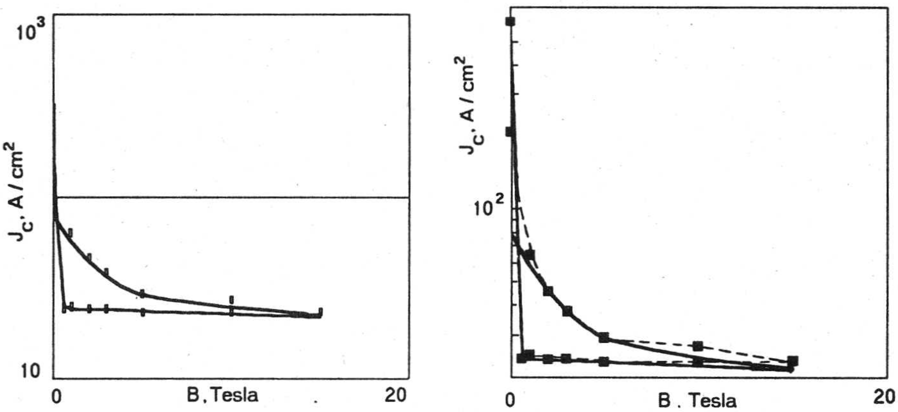
i.e. $\Gamma\kappa/\ln(\kappa) >> 1$. Here we used the anisotropic Ginzburg–Landau theory. It is assumed that the external field H_e is parallel to *ab* plane and *c* axis is perpendicular to the Josephson junction area, $\Gamma = \lambda_c/\lambda_a$, $\kappa = \lambda_a/\xi_a$. Vortex-lattice decoration experiments (Vinnikov et al. [22]) have shown that in YBCO $\Gamma = \lambda_c/\lambda_a \cong 5$, $\kappa \cong 100$, so $H_s >> H_{c1}$.

3. Comparison to experimental data

We use our model to interpret some experimental data on J_c vs H hysteresis obtained for YBCO–123 ceramics [15,16]. Similar results have been qualitatively interpreted in terms of an effective hysteretic magnetic field at the grain interfaces [16,23]. In our model the effective magnetic field at the grain surface is assumed to be equal to the external field H_e, i.e. we avoided the complication of demagnetizing factors from the grains. Of course, if the ceramics granules are in the Meissner phase, the magnetic force lines can be concentrated substantially near the Josephson junctions even at small demagnetizing factor of individual granules. The analysis of this problem is in the general case rather difficult since the result depends on the intergrain cluster topology, granule shape and orientation, etc. There are, however, rather general considerations according to which the local field effect (if it really makes a considerable contribution in the case of this particular specimen) is exhibited in the $J_c(H)$ dependencies.

Let us consider, for example, highly textured ceramics with plate-like granules having thickness L. Let the external magnetic field be directed parallel to the texturing *ab*-plane (i.e. the plate surface) and normal to the transport current. In the Meissner phase a part of the flux is expelled into the intergrain space, and the necessity to maintain the full magnetic flux results in an increase of the local magnetic field by about $L/(2\lambda_a)$ times. So the respective field acting on the contacts of the medium is $H_{eff} \approx L/(2\lambda_a)H_e$. If the full flux penetration field is small ($H_p \equiv j_{cg}L << H^g_{c1}$) the increase of the local fields will result in somewhat earlier penetration of the magnetic flux into the granules and the field dependence of the ceramics critical current $J_c(H)$ should exhibit the bend due to both sharp weakening of local fields at intragranular flux diffusion and the change in the nature of the Josephson junction response to the field in the presence of the Abrikosov vortices.

If the intergrain cluster is so rare that the magnetic field goes into its cells freely, the local increase of the magnetic field becomes less pronounced. That cluster have been realized in early works on YBCO–123 ceramics [15,16]. In Fig. 1 *a,b* the experimental results [15,16] are compared with our calculations of J_c vs H depend-



a *b*

Fig. 1. The experimental (squares) and calculated (solid line) hysteresis loop of the transport critical current density for YBCO metal ceramic: (*a*) experiment Watanabe et al. [15], (*b*) experiment Kwasnitza et al. [16]); *P*, field increase, *p*, field decrease

ence (see below). The most important feature of these curves is a power law $J_c \sim 1/B^{1/2}$ dependence of the critical currents for decreasing field. Here the J_c vs H dependence is approximately

$$J_c(B_\downarrow) \approx \frac{J^\downarrow_{c0} B_{jc2}}{B_{jc2} + \sqrt{2B_{c1}B_\downarrow}}, \qquad B_{jc2} = \Phi_0/(4\pi\lambda_a r)\,. \tag{15}$$

The $J_c(B_\downarrow)$ value (15) was obtained from the equations

$$H_{ex}(B_\downarrow) = (B_\downarrow)/\mu_0 + H_{c1}, \quad J_c(B_\downarrow) \approx J_{c0}/(1 + |\Delta\varphi_\downarrow|),$$

$$\Delta\varphi_\downarrow = r\left[2\pi\mu_0/\Phi_0\left(2\lambda_a^2 j^\downarrow_{sf} + d_N H\right)\right], \quad j_{sf} \approx \frac{\left[H^2 - (B_\downarrow/\mu_0)^2\right]^{1/2}}{\lambda_a}. \tag{16}$$

There are two main features in hysteretic phenomena in decreasing field $H_\downarrow$. Namely, the depression of the critical current at $H = 0$, when

$$\Delta\varphi = \Delta\varphi_{rem} = r\left[2\pi\mu_0/\Phi_0\left(2\lambda_a^2 j_{cg}\right)\right],$$

(13), therefore

$$J_c(0)^\downarrow / J_c(0)^\uparrow \cong J_c(H^*_{rem})^\uparrow / J_c(0)^\uparrow < 1\,,$$

where the "effective remanent magnetic field H^*_{rem}" is

$$H^*_{rem} = \lambda_a \cdot j_{cg}, \tag{17}$$

and a shift ΔH of maximum J_c vs. H position. According to (12) in our model the maximum of Josephson critical current in decreasing field must be observed in the vicinity of field $H \cong \Delta H$, where

$$\Delta\varphi \sim j_{sf}(\Delta H) \cong M^{ex}(\Delta H) - j_{cg} \cong 0. \tag{18}$$

If $H_{c1} > \lambda_a j_{cg}$ then $M^{ex}(H) = H$ and the field $\Delta H = j_{cg}(\Delta H)\lambda_a$, so the ΔH value is equal to the effective magnetic field $H^* = \lambda \cdot j_{cg}$ (17). In another limit $H_{c1} << \lambda_a j_{cg}$

$$\left[2\Delta H H_{c1}\right]^{1/2} \approx \lambda_a j_{cg}(\Delta H). \tag{19}$$

Here H_m is the maximum of external field H_m, the condition $H_m \geq H_{c1}$ is assumed. According to (19) in the case $H_{c1} < \lambda_a j_{cg}$ for fields $H_{c1} < H_e < H_p + H_{c1}$ the ΔH value depends on the maximum field H_m.

For the analyzed superconductor and for the chosen Josephson junction the values λ_a, Γ, ξ_a and d_N are given, so the essential parameters are the granular critical current j_{cg} and the correlation radius r of the spatial fluctuations of the Josephson critical current (or the width of small junctions). (The J_{c0} value depends on the structure of the percolation cluster in the Josephson medium, and it was chosen so that $J_c^{exp}(0) = J_c^{calc}(0)$.) But the H_s value given by $H_s = \Phi_0/(4\pi\mu_0\lambda_a\xi_a)$ is not strictly correct [21,24,25] so we use a more general form

$$H_s = q\Phi_0/(4\pi\mu_0\lambda_a\xi_a), \tag{20}$$

where q is a constant of the order of unity. It was found that the calculated hysteresis loop (Fig. 1) is very sensitive to the q value, while the parameters r and j_{cg} determine mainly the plateau region length H_{j2} (14) (for $H_p \leq H_\downarrow \leq H_{j2}$ the $J_c(H_\downarrow)$ value is almost constant). Parameter q determines the value of magnetic field $B = B_\xi \equiv (q^2/4 \ln\kappa)B_{c2}$, when $M_{en} \approx M_{ex}$ (7,8), i.e. hysteresis of $J_c(H)$ caused by the surface barrier is absent for $B > B_\xi$. As a result, for given j_{cg} value parameter q determines the critical current J_c amplitude.

The calculated curves describe the qualitative features of the data [15,16] quite well (Fig. 1). The critical current J_c amplitude value as well as the behaviour of the J_c vs. H dependence for decreasing external fields are well represented. It is evident that egs. (7)−(13) give a good fit to the data for parameter $q = 0.4 \div 0.5$. In our opinion it is convincing evidence, that in case of the experiment [15,16] the J_c vs. H hysteresis is caused by the surface barrier irreversibility.

The temperature dependence of the hysteresis in YBCO ceramics comes from the studies by Watanabe et al. [15]. The hysteresis becomes smaller with increasing of temperature, but still remains clearly at a lower field region. When there is a potential barrier the temperature dependence of the critical current can be described by

$$J_c(T) \approx J_{c0}(T) B_r / (B_r + |B_{eff}^{\downarrow\uparrow}(T)|),$$

where for increasing field

$$B_{eff}^{\uparrow}(T) = \frac{2\lambda_0}{d_N}\left[B_{s0} t^{1/2} + \lambda_a^0 \mu_0 j_{cg}^0 t^{M-1}\right] + B, \tag{21}$$

and for decreasing field

$$B_{eff}^{\downarrow}(T) \approx \frac{2\lambda_0}{d_N}\left[(2B_c^0 B)^{1/2} - \lambda_a^0 \mu_0 j_{cg}^0 t^{M-1}\right] + B. \tag{22}$$

Here $t = 1 - T/T_c$, $B_r = \Phi_0/[2\pi r d_N]$, the exponent M arises from the granular critical current temperature dependence $j_{cg}(T) = j_{cg}^0 t^M$. According to (21)–(22) the hysteresis value depends on the difference between $B_{eff}^{\downarrow}(T)$ and $B_{eff}^{\uparrow}(T)$ and decreases for $T \to T_c$ if the exponent $M > 1$ (see Fig. 2). The characteristic field B_ξ above which the hysteresis amplitude is negligible is decreasing also: $B_\xi = B_{\xi 0} t$, $B_{\xi 0} = B_{s0}^2 / 2B_{c1}^0$ $(B >> B_{c1}^0)$.

The similar result can be obtained in the absence of a surface barrier

$$B_{eff}^{\uparrow} = \frac{2\lambda_0}{d_N}\left[B_{c1}^0 t^{1/2} + \lambda_a^0 \mu_0 j_{cg}^0 t^{N-1}\right] + B, \tag{23}$$

$$B_{eff}^{\downarrow} \approx \frac{2\lambda_0}{d_N}\left[B_{c1}^0 t^{1/2} - \lambda_a^0 \mu_0 j_{cg}^0 t^{N-1}\right] + B, \tag{24}$$

but here the abrupt suppression of the hysteresis J_c value is absent.

4. Discussion

The critical current of a Josephson junction depends on the full phase difference $\Delta\varphi = \Delta\varphi_{sf} + \Delta\varphi_H$ along the junction width W, which consists of the two parts. One is connected with the surface current value, $\Delta\varphi_{sf} \equiv [2\pi\mu_0/\Phi_0]\, W 2\lambda_a^2 j_{sf}$, and another is due to the external field H_e penetration to the intergranular space $\Delta\varphi_H \equiv [2\pi\mu_0/\Phi_0]\, W d_N H_e$. If the junctions length d_N is small enough, so that $\Delta\varphi_H << \Delta\varphi_{sf}$, then behaviour of the Josephson critical current in a magnetic field is determined mainly by the surface current j_{sf} value. It depends on the jump of the magnetic field on the granular surface.

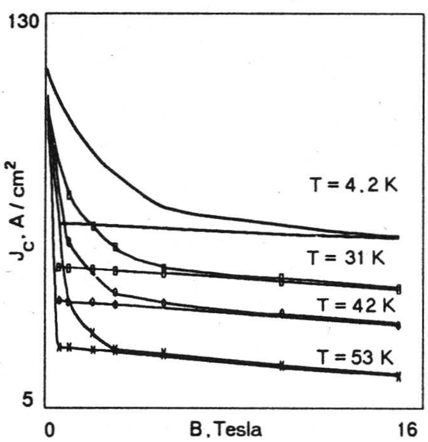


Fig. 2. Calculated temperature dependence of hysteresis for YBCO (experiment see in [15])

If there is the equilibrium connection between the external field H_e and induction B at the surface (2),

$$H_e \cong B(0)/\mu_0 + M_{eq},$$

then $j_{sf} \approx H_{c1}/\lambda_a$. In the presence of a barrier against flux entry or exit the boundary conditions on H_e , where flux has last entered is $H_e = H_{en}$, and the conditions where flux has last exited is $H_e = H_{ex}$. In the numerical calculations involving the critical entry and exit fields, the following two models so far have been used: (i) the no-entry and no-exit barrier model (2), and (ii) the square root model (10)–(11). But the total scheme for the surface currents calculation (7)–(9),(12) allows one to use any model.

It must be born in mind that these models should not be taken too seriously, since they do not have a solid theoretical foundation. The most difficult is to understand the possibility of fulfillment of the equilibrium boundary conditions (2), at least this condition is widely used [17,24]. The point is that for these condition the distance between the surface and nearest vortex x_0 does not coincide with the equilibrium value of $x_0 = x_v + h_a/2$ (6). As a consequence, one can think that there exists an additional Lorentz force which repulsed vortices from the surface of superconductor. The influence of that force may be compensated by a supplementary compression of a vortex lattice.

But the straightforward numerical calculations of the Gibbs free energy for the vortex structure shows that a highly small compression of the distance between the vortices in the direction parallel to the surface of the slab gives the equilibrium position of x_0 corresponding to the boundary conditions (2).

It should be noted that the surface current value j_{sf} is very sensitive to the distance x_0 and for the "equilibrium" boundary conditions (2) the equality

$$\exp\left[(x_0 - h/2)/\lambda_a\right] = x_\lambda/\sinh(x_\lambda) \tag{25}$$

must be fulfilled. Only in that case the j_{sf} value is $j_{sf} \approx H_{c1}/\lambda_a$. Near the surface the current density is

$$j^{\uparrow\downarrow}(x) = \frac{1}{\lambda_a}\left[M_{en(ex)}\exp(-x/\lambda_a) - \left[H - M_{en(ex)}\right]\sinh(x/\lambda_a)\right], \tag{26}$$

$x < x_0$. Here the first term is a modified Meissner part and the second one is the contribution of vortices.

If the condition (25) is violated an additional square root dependence of the surface current exists: $j_{sf} \sim B^{1/2}$ (see, for example, (15)). It leads to the well-distinguished dependence of the Josephson media critical current $J_c(B) \sim B^{1/2}$. Exception appears in the case of the surface barrier existence, when the $j_{sf}^{\downarrow}$ value reaches its maximum probable value $j_{sf}^{\downarrow} \approx H_s/\lambda_a$ (7),(10), close to the Ginzburg–Landau critical current j_c^{GL}. Analysis of a great number of works, where such $J_c \sim H^{1/2}$ dependence has been observed shows that it may be connected with the field dependencies of the intragranular critical current j_{cg} rather than with the intergranular one (the polycrystal films and the case when the normal component of the external magnetic field leads to the strong J_c vs H dependence (Tachiki and Takahashi [26], Matsushita et al. [27]). In any event if the plateau region is strongly pronounced then J_c vs. H dependence in that region has roughly linear character, $J_c \sim H$ (see, for example, [28]).

For decreasing fields, a power low $J_c(H_\downarrow) \sim H^{-N}$ with N ranging between 0.6 and 2.8 has been observed by Deutscher et al. [29]. The plateau region in that work shows a maximum which may be connected with j_{cg} vs. B dependence (see eqs. (1), (9), (12)). As is shown above the $J_c(H_\downarrow) \sim H^{-1/2}$ takes place for the Watanabe et al. [15] and Kwasnitza et al. [16] experiments as well. In accordance with the presented theory this evidences about the surface barrier in the investigated samples.

But where may the surface barrier in the conventional nontextured metal ceramics appear from [15,16]? On the other hand, what supports the strained vortex state which is needed for satisfaction of the equilibrium boundary condition $H_e \cong B(0)/\mu_0 + M_{eq}$ (2)? From our point of view the answer to these questions cannot be obtained within the bounds of the conventional equilibrium theory based on the Gibb's free energy minimization [24].

A limited nature of the equilibrium approach is evident if one takes into account that the critical state of type II hard superconductors is nonequilibrium in principle and its current state is metastable. Therefore the flux creep (flux diffusion) process plays an important role in the process of the critical state establishment.

The existence of pinning centers leads to dependence of flux distribution on the magnetic history of the sample. The critical state model (Bean [30], London [31]) assumes that, when a current or field is changed in a specimen, shielding currents are induced on the surface up to the maximum density J_c. When this is reached the current density remains constant, and magnetic flux penetrates deeper into the superconductor. According to the critical state model the J_c may depend on the local microstructure of specimen and flux density, but not the experimental situation. The justice of this model is given by enormous amount of the experimental facts [24]. The microscopic interpretation of the critical state model is based on the idea of flux creep and well-known nature of the driving force on vortices [32].

The vortices had been nucleating at the surface and moving into the superconductor until the force due to the gradient density of vortices was balanced by the pinning (here it is assumed that vortices do not nucleate in the volume of the sample). The flux distribution for any cycle of external field or transport current is then defined uniquely by the pinning force. The flux density is usually assumed to take its equilibrium value at the surface, as defined by the reversible magnetization curve ($H_e \cong B(0)/\mu_0 + M_{eq}$). Deviations from these conditions are called surface barriers.

Additional physical considerations are necessary for a proper choice of the boundary conditions. Here I use the principle of minimum entropy production (minimum dissipation [33]). According to this principle, a regime with the minimum possible value $Q = \mathrm{jE}$ should be realized on the surface. This choice is equivalent to the condition that the surface supercurrent j_{sf} which is responsible for the vortex injection, should have the minimum possible value. For a nonuniform surface the Meissner state becomes unstable (for $H_s >> H_e > H_{c1}$) at the surface defects, where surface current density is close to j_c^{GL}. Then, under the influence of the surface current $j_{sf} \approx j_{eq} \approx H_{c1}/\lambda$ (out of the surface defects) the Abrikosov vortex germ broadens over the full surface region. In an ideal material flux lines would distribute homogeneously across the whole sample according to the equilibrium condition (2). For a hard superconductor, however, this is inhibited by the pinning forces which allow a vortex movement only as long as the driving current density $j > j_{cg}$.

Consequently, almost on the whole sample surface the condition $j_{sf} \approx j_{eq}$ is fulfilled, which minimizes the dissipation in time of the Abrikosov vortex line nucleation. For that reason the observed coincidence of j_{sf} value width $H_s/\lambda \approx j_c^{GL}$ (Sec 2) may be connected with "weak places" (surface defects) on the superconducting surface. From these considerations the present author assumes, that in the given experiments [15,16] almost all Josephson weak links are simultaneously the "weak place" for Abrikosov vortex nucleation in the superconductor electrodes, so in their region the surface current reaches its maximum possible value $j_{sf} \approx j_c^{GL}$. It is equivalent to fulfillment of the surface barrier boundary conditions (10,11) in those weak places.

5. Conclusions

In the framework of the discussed model the reason for J_c vs. H hysteresis lies in the irreversible dependence of the granule surface current. The critical current J_c stability to high magnetic fields may be due to both the inhomogeneity of the Josephson weak links (small Josephson junctions width r value (4),(14)) and existence of the Abrikosov vortices within the granules.

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