Communications

SPECTRUM OF COLLECTIVE MODES IN A THICK FILM OF A *p*-WAVE SUPERCONDUCTOR

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Introduction. The present work attempts to find a signature for "exotic" pairing in a superconductor. In principle, it is possible to identify the type of pairing from measurement of the heat capacity, but it is difficult to identify the type of pairing unequivocally, because of the close similarity of the behaviour of the heat capacity for the cases of *p*- and *d*-pairing.

We propose a new criterion: to identify the pairing scheme by studying the lifting of degeneracy of the collective-mode spectrum. It is well known that this spectrum depends on the kind of pairing present. For systems with many internal degrees of freedom , the spectrum will be more complicated than for a simple *s*-wave paired BCS superconductor; indeed the number of degenerate modes will be equal to the number of components of the order parameter [1]. External perturbations can break the degeneracy of the collective-mode spectrum, and display its properties more clearly. We have therefore studied the spectrum of a thick *p*-wave paired heavy-Fermion superconducting film [2]; the boundaries introduce a degeneracy-breaking perturbation. This leads to the formation of "textures", analogous to those seen in ^{3}He . The resulting degeneracy breaking displays the internal properties of the Cooper pairs more clearly than is possible in the degenerate situation. We find eight collective modes (i.e. not *all* the degeneracy is lifted), whose frequencies depend on the direction of the excitation. Most of the modes have an energy gap.

We also find that the temperature dependence of the collective-mode spectrum can provide a sensitive probe of T_c. For example, ultrasound is absorbed by pair-breaking collective modes; this resonant absorption of ultrasound will peak sharply at the onset of superconductivity.

Model. Volovik and Gor'kov [3] make a two-dimensional Ansatz for the order parameter of a cubic superconductor (for pair spin $S = 1$), i.e. *p*-pairing:

$$\mathbf{d}(\mathbf{k}) \sim \hat{\mathbf{x}} k_x - \hat{\mathbf{y}} k_y , \qquad (1)$$

We define a tensor order parameter A_{ij}, and write

$$d_i(\mathbf{n}) = A_{ji}n_j, \quad \text{where } n_i = k_i/|k|$$

and i and j are respectively spin and orbital indices. The Volovik-Gor'kov form (1) requires that

$$A_{ji} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For a film whose thickness is rather larger than the coherence length, the gradient term of the energy may become much larger than the spin-orbit (i.e. dipole-dipole) energy. This enables us to take the following simple Ansatz for the order parameter between the middle of the film and the boundary:

$$\Delta_{ia} = \Delta \left[f(x) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + (1 - f(x)) \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

$$\Delta_{ia} = \Delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 + f(x) & 0 \\ 0 & 0 & -f(x) \end{pmatrix}. \tag{2}$$

Here the gap function $\Delta = \Delta_0 \sin\theta$ has zeros at the two points $\theta = (0,\pi)$ — i.e. at the intersections of the Fermi surface with a four-fold axis of the cubic crystal [1]. We have previously shown [4] that in a Bose system with p-pairing (*cf.* ^{3}He), the order parameter matrix takes the form

$$\begin{pmatrix} \Delta_1 & 0 & 0 \\ 0 & \Delta_2 & 0 \\ 0 & 0 & \Delta_3 \end{pmatrix}. \tag{3}$$

The general quadratic form for the action functional is:

$$\begin{aligned} \sum_p (u_{ia}u_{jb} - v_{ia}v_{jb}) \Big[& \Delta_{ia}(q)\Delta_{jb}(q') + \Delta_{ja}(q)\Delta_{ia}(q') + \Delta_{jb}(q)\Delta_{ia}(q') + \\ & + \left(\Delta_{kb}(q)\Delta_{ka}(q') + \Delta_{ka}(q)\Delta_{kb}(q')\right)\delta_{ij} - \big(\Delta_{ia}(q)\Delta_{ja}(q') + \Delta_{ja}(q)\Delta_{ia}(q') + \\ & + \Delta_{ka}(q)\Delta_{kb}(q')\delta_{ij}\big)\delta_{ab} \Big] 2B(p) - \sum_p (u_{ia}u_{jb} + v_{ia}v_{ja})\delta_{ij} 5A(p). \end{aligned} \tag{4}$$

For the particular gap matrix (3), the coefficients take the form

$$A \sim \tilde{\Delta}^2 + \tfrac{1}{2}\omega^2; \quad B \sim -\Delta^2; \quad \tilde{\Delta}^2 = n_{1i}n_{1j}\Delta_{ia}\Delta_{ja}.$$

Substituting (2) in (4), we find three distinct independent quadratic forms. Two of them are three-dimensional, for the variables

$$(u_{11},u_{22},u_{33}) \text{ and } (v_{11},v_{22},v_{33}) \, , \tag{5}$$

and the third is a two-dimensional form for the remaining variables

$$\begin{array}{ccc} (u_{12},u_{21}) \, , & (u_{23},u_{32}) \, , & (u_{13},u_{31}) \\ (v_{12},v_{21}) \, , & (v_{23},v_{32}) \, , & (v_{13},v_{31}) \end{array} \tag{6}$$

Here the u and v are respectively the real and imaginary components of a complex Bose field.

Results. The solutions of the above quadratic forms determine the spectrum of collective modes. We must solve the equations

$$\det Q_i = 0$$

where Q_i are the quadratic forms. We find the following results for the frequencies of the collective modes:

$$\omega^2 = 0 \, , \quad \omega^2 = 3\Delta^2$$

for the variables (6), and

$$\omega_i^2 = \alpha_i \Delta^2 \text{ for the variables } (u_{11},u_{22},u_{33}) \, ,$$

$$\omega_i^2 = \beta_i \Delta^2 \text{ for the variables } (v_{11},v_{22},v_{33}) \, ,$$

where all the six distinct coefficients α_i, β_i are constants, of order unity.

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4. *Brusov P. N. and Bukshpun M. M.*, Physica B165-166, 685 (1990).