SUPPRESSION OF FLUCTUATIONS IN JOSEPHSON JUNCTIONS

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An approximate analytical expression is derived permitting to predict fluctuation across overdamped Josephson junctions and, thus, take measures for its suppression in electron devices using the Josephson effect. It is shown that maxima in the spectral density of noise and signals are at $\omega \simeq 2f$, where ω is the signal recurrence and f is the recurrence of fluctuation jumps of voltage.

It's well known that the Josephson effect has been used in different electronic devices. To provide stable operation of these devices a problem of suppression of fluctuations must be solved. A Josephson junction can be regarded as a system with two states: superconductive and resistive states. The existence of two stable states and random transitions from one to another can be regarded as a random telegraph signal. If there are both random telegraph signal and periodic signal, then the system undergoes the stochastic resonance (SR): increase of input noise can improve the signal-to-noise ratio.

In this paper we give an expression permitting to predict such phenomena in overdamped Josephson junctions. It should be outlined that in this case (nonhysteresis regime) the system isn't bistable.

In this paper we have calculated the fluctuation spectrum for voltage of Josephson contacts (JC) and we have used the following approximations:

a) signal appearance is much less in comparision with the rate of relaxation (adiabatic approximation);

b) it's assumed that the rates of random jumps are known. The current fluctuations are small and the Kramers' formula for transition rates is true.

These assumptions and knowledge of the periodic potential allow us to use the theory [1] for bistable systems. We have found a range of parameters, at which SR may be observed.

The equation for the dynamics of an undimensional JC with small capacitance has a form [2]:

$$\left.\begin{aligned} &\gamma\dot{\varphi} + \sin\varphi = i_0 + i_1 \sin\Omega_0 t + \xi(t) \\ &<\xi(t)\xi(t')> = 2\pi\eta\delta(t-t') \end{aligned}\right\} \tag{1}$$

$$\begin{aligned} &i_1 = I_1/I_c; \quad i_0 = I_0/I_c; \quad \omega_j = (2eI_c/\hbar c)^{1/2}; \\ &\eta = \frac{ekT}{2\hbar I_c}; \quad \Omega_0 = \omega/\omega_j; \quad \gamma = (RC\omega_j)^{-1} \end{aligned} \tag{2}$$

where φ — phase differences between neighboring superconductors; R, C — resistance and capacitance; i_0 represents the normalized dc bias current of periodic signal; time is normalized with respect to the inverse of the Josephson plasma frequency ω_j and current is normalized to the Josephson critical current I_c.

The stochastic equation (1) describes the motion of a particle in the periodic potential

$$U(\varphi) = -\cos\varphi - i_0\varphi .$$

For calculation of probability distribution $W(\varphi,t)$ used by us that potential $U(\varphi)$ is periodic and, therefore, we can limit the interval $[0, 2\pi]$ for φ [3] and, then, use the approximation of random walking [1]. In this approximation $U(\varphi)$ has only two minima and for $W(\varphi,t)$ it may be written:

$$W(\varphi,t) = W_1(t)\delta(\varphi - \varphi_1) + W_2(t)\delta(\varphi - \varphi_2),$$

where φ_i $(i = 1, 2)$ are the points of potential minima $U(\varphi)$:

$$\varphi_1 = \sin^{-1} i_0; \quad \varphi_2 = 2\pi + \sin^{-1} i_0 .$$

And for $W_i(\varphi,t)$ it may be written:

$$\frac{dW_i}{dt} = -r_i(t)W_i + r_j(t)W_j; \quad W_1 + W_2 = 1; \quad i, j = 1, 2, \tag{3}$$

r_i is the transition rate from φ_i to $\varphi (i \neq j)$.

The common solution for equations (3) may be presented in the following form

$$W_1(t) = g^{-1}(t)\left\{ W_1(t_0)g(t_0) + \int_{t_0}^{t_1} r_2(z)g(z)dz \right\},$$

$$g(t) = \exp\left\{ \int_{t_0}^{t_1} \left(r_1(z) + r_2(z)\right) dz \right\}. \tag{3a}$$

We must interprete as probability the fact that the phase differences become more than $|\pi|$. It means, that the voltage is not equal zero through the contact.

If there is no signal, the Kramers' approximation gives such expressions for transition rates:

$$r_1 = \frac{\sqrt{1 - i_0^2}}{2\pi\gamma} \exp\left\{ -2 \frac{i_0 \cos^{-1} i_0 + \sqrt{1 - i_0^2}}{\eta} \right\},$$

$$r_2 = r_1 \exp\left(-\frac{2 i_0 \pi}{\eta} \right). \tag{4}$$

The transition rates as the function of time can be calculated, if the time of relaxation is less than the period of the signal: $(2\pi\gamma)/\sqrt{1 - i_0^2} >> \Omega_0$. The last unequality is equivalent to

$$RC << 2\pi/\omega . \tag{5}$$

The condition (5) allows us to consider that the potential form is not changed. Only the transition rate is modulated by the periodic signal.

We shall investigate only the symmetric case ($i_0 = 0$):

$$r_1 = r_2 = r = \frac{1}{2\pi\gamma} \exp(-2/\eta); \tag{6}$$

$$r_1(t) = \frac{1}{2\pi\gamma} \exp\left\{\frac{2}{\eta}(-1 + i_1 \cos \Omega_0 t)\right\}$$

$$r_2(t) = \frac{1}{2\pi\gamma} \exp\left\{\frac{2}{\eta}(-1 - i_1 \cos \Omega_0 t)\right\} \quad (7)$$

Using the solution for W_i (3) and expressions for $r_i(t)$, we can obtain expressions for the conditional probabilities $W_i(t/\varphi_j(t_0))$ and, hence, to obtain the expression for the autocorrelation function. Indeed, for the autocorrelation function such formula is true:

$$K(\tau) = \sum \varphi_i \varphi_j W_i(t + \tau/\varphi_j(t)) W_{is} - <\varphi>^2 \quad (8)$$

where $W_{i,j}$ is the stationary probability. It may be obtained from $W_i(t/\varphi_j(t_0))$ if $t_0 \to -\infty$.

The power spectrum $S(\Omega)$ is given by [4]:

$$S(\Omega) = 4 \int_{-\infty}^{\infty} K(\tau) \cos \Omega t d\tau .$$

The power spectrum of voltage fluctuations may be found by the expression for voltage on the contact $V = \frac{\hbar}{2e}\dot{\varphi}$, hence, $S_V(\Omega) = S(\Omega) \cdot \Omega^2$. Finally, the power spectrum $S_V(\Omega)$ is given by the expression:

$$S_V(\Omega) = 8\pi^2\Omega^2 \Big\{ \left[r/\left(4r^2 - \Omega^2\right) \right] \times \left[1 - 2r^2 (i_1/\eta)^2 \right] / \left[\left(4r^2 + \Omega_0^2\right) \right] +$$

$$+ \left[2\pi^2 r^2 (i_1/\eta)^2 \right] / \left[4r^2 + \Omega^2 \right] \cdot \delta(\Omega - \Omega_0) \Big\}. \quad (9)$$

It's seen that the power spectrum can be divided into two parts: 1) the broadband output noise S_N ; 2) the output signal S_S :

$$S_V(\Omega) = S_N(\Omega) + S_S(\Omega) .$$

It's seen that the value for suppression of noise exceeds the frequency of signal appearance and the signal amplitude is more.

From expression (9) we have obtained the signal-to-noise ratio:

$$R(\eta) = S_S(\Omega)/S_N(\Omega) ,$$

$$R(\eta) = \gamma^{-1} \left(e^{-1/\eta} \cdot (i_1/\eta)\right)^2 \left[1 - \frac{e^{-4/\eta} \cdot (i_1^2/\eta^2)}{2\left(e^{-4/\eta} + \Omega_0^2 \pi^2 \gamma^2\right)}\right]^{-1} . \quad (10)$$

The maximum value $R(\eta)$ may be expected at $\eta \sim 1$. The "real" resonance must be connected with the transition rate. Indeed, maxima $S_S(\Omega)$ and $S_N(\Omega)$ are given by the next relations between the signal frequency and the coefficient η :

for the signal: $\Omega^2 = \frac{\eta \cdot e^{-4/\eta}}{2 - \eta} (\pi\gamma)^{-2} = 8r^2/(2 - \eta)$;

for the noise: $\Omega = (\pi\gamma)^{-1} e^{-2/\eta} = 2r$.

It's seen that both signal and noise spectra reach their peak, if the signal frequency equals the rate, at which current fluctuations exceed the critical current I_c.

Let us see now, what must be parameters JC to observe SR. For $\eta \sim 1$ in normal superconductors $I_c \sim 10^{-3}-10^{-5}$ A and $\eta \sim 10^{-3}-10^{-1}$ T. Hence, even in normal superconductors for $T_c > 10$ K the maximum of fluctuations suppression may be observed.

If capacitance $C \sim 10^{-10}-10^{-12}$ F, then Josephson plasma frequency is $\omega_j \sim (10^{10}-10^{11})$ sec^{-1}. Nearly resonance undimensional rate approximately equals 10^{-2}. It means that to observe the peak signal-to-noise ratio the frequency of *ac* must follow the relation: $\omega/\omega_j << 10^{-2}$, i.e. $\omega << (10^8-10^9)$ sec^{-1}.

The resonance increase of both noise and signal spectra can be observed for frequency $\Omega_0 \approx 2r$, i.e. $\omega/\omega_j \sim 2\cdot 10^{-2}$.

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