ON DISTRIBUTIONS OF NONEQUILIBRIUM ELECTRON QUASIPARTICLES IN ANISOTROPIC SUPERCONDUCTORS WITH NON-DEBYE PHONON SPECTRUM

A. M. Gulian, H. N. Nersesyan, S. V. Paturyan

Institute for Physical Research, Armenian Academy of Sciences, Ashtarak-2, 378410, Armenia

Assuming a "two-peak"-character phonon density of states in superconductors, it is shown that the deviation of electron system from its equilibrium state may become substantially greater compared with a well-studied ideal model of metals. If charge carriers interact with transverse phonon modes, then nonequilibrium deviation is increased additionally and the creation of inversely populated states becomes possible.

Introduction. For more than two decades the possibility of inverse electron population in superconductors attracts attention as it is of importance for solving of various problems, such as superconductivity at repulsion between electrons [1,2], paramagnetic current response [3], instabilities of an order parameter collective oscillation modes [4], phonon [5,6] and photon [7,8] instabilities and so on.

Considering the problem of inverse population, we are bearing in mind such a state of the electron system, at which the distribution function n_ε fulfills the condition

$$n_\varepsilon > 1/2 \qquad (1)$$

for some values of electron excitation energies lying in the overgap region $\varepsilon \geq \Delta$ (hereafter we consider symmetrically populated electron-hole branches only, neglecting branch imbalance, see, e.g. [9], Δ is the superconducting gap in energy spectrum of electrons). A pumping by electromagnetic radiation is one of the most appropriate techniques for preparation of SC samples with nonequilibrium states [10]. Typically the gap value Δ is significantly smaller than the value of pumping electromagnetic field frequency ω_0 (the energy scale): $\omega_0 >> \Delta$. Despite of many experiments exploring this technique (see, e.g., [11-13]), up to now inversely populated states are not obtained.

Moreover, a theoretical investigation [14] has given arguments (seemingly strong) in favour of principal impossibility of inversely populated states in superconductors under the pumping of optical radiation.

Nevertheless, the main goal of the paper presented is to show that the model used in [14] in some respects oversimplify the problem and there may be real cases at which the inequality (1) is to be fulfilled.

Basic equations. We shall consider the high-frequency electromagnetic field influencing a thin SC film. The film is assumed to have a thickness $d < v_F/T_c$ so that the "phonon-bath" model as well as the kinetic equations of Eliashberg [15] may be applied. Thus the equation for quasistationary distribution function of electron excitation in the spatially homogeneous case may be presented in the form

$$0 = Q(\varepsilon) + \int_\Delta^\infty \frac{\varepsilon\varepsilon' d\varepsilon'}{\sqrt{\varepsilon^2 - \Delta^2}\sqrt{\varepsilon'^2 - \Delta^2}} \int_0^\infty d\omega_q \alpha^2(\omega_q) F(\omega_q) \times$$

$$\times \Big\{ \left(1 - \eta \frac{\Delta^2}{\varepsilon\varepsilon'}\right) \left[n_{\varepsilon'}(1 - n_\varepsilon)(N_{\omega_q} + 1) - (1 - n_{\varepsilon'}) n_\varepsilon N_{\omega_q} \right] \delta(\varepsilon' - \varepsilon - \omega_q) +$$

$$+ \left(1 - \eta \frac{\Delta^2}{\varepsilon\varepsilon'}\right) \left[n_{\varepsilon'}(1 - n_\varepsilon) N_{\omega_q} - (1 - n_{\varepsilon'}) n_\varepsilon (N_{\omega_q} + 1) \right] \delta(\varepsilon' - \varepsilon + \omega_q) +$$

$$+\left(1+\eta\frac{\Delta^2}{\varepsilon\varepsilon'}\right)\left[(1-n_\varepsilon)(1-n_{\varepsilon'})N_{\omega_q}-n_\varepsilon n_{\varepsilon'}(1+N_{\omega_q})\right]\delta(\varepsilon'+\varepsilon-\omega_q)\Big\}\,, \quad (2)$$

where $Q(\varepsilon)$ is the source of nonequilibrium electron excitations:

$$Q(\varepsilon)=\alpha_0\left[U_-(n_{\varepsilon-\omega_0}-n_\varepsilon)-U_+(n_\varepsilon-n_{\varepsilon+\omega_0})+V(1-n_\varepsilon-n_{\omega_0-\varepsilon})\right], \quad (3)$$

and also $\alpha_0=D\left(\frac{e}{c}\right)^2 A_{\omega_0}A_{-\omega_0}$ is the parameter proportional to the intensity of incident electromagnetic field (D is the diffusion coefficient, A_{ω_0} is the amplitude of the field vector-potential, $\hbar=1$), the factors $U_\pm$ and V are defined as

$$U_\pm=\frac{[\,\varepsilon(\varepsilon\pm\omega_0)+\Delta^2]\,\Theta(\varepsilon\pm\omega_0-\Delta)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\varepsilon\pm\omega_0)^2-\Delta^2}}\,,$$

$$V=\frac{[\,\varepsilon(\omega_0-\varepsilon)-\Delta^2]\,\Theta(\omega_0-\varepsilon-\Delta)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\omega_0-\varepsilon)^2-\Delta^2}}\,, \quad (4)$$

the function $N_\omega=\left(\exp\frac{\omega}{T}-1\right)^{-1}$ describes the equilibrium (in the "phonon-bath" model) phonon system at the temperature T. A parameter η in the Eq. (2) is unity for the case of longitudinal phonons and -1 for the case of transverse phonons (in general case the summation over the polarizations of phonons must be undertaken in Eq. (2), but we shall restrict ourselves to the case of phonon field with a definite polarization).

Simple analytical solution. First of all we shall present here the results of the simplified analytical consideration for $T=0$. Following Yelesin [14], Eq.(2) may be presented in a form:

$$-(1-n_\varepsilon)S^+(\varepsilon)+n_\varepsilon S^-(\varepsilon)+n_\varepsilon S^R(\varepsilon)=Q(\varepsilon), \quad (5)$$

where ($\eta=1$):

$$\begin{pmatrix}S^+\\S^-\\S^R\end{pmatrix}=\int_\Delta^\infty d\varepsilon'\int_0^{\omega_D}\frac{d\omega_q\,\varepsilon\varepsilon'\alpha^2(\omega_q)F(\omega_q)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{\varepsilon'^2-\Delta^2}}\begin{pmatrix}n_{\varepsilon'}\delta(\varepsilon'-\varepsilon-\omega_q)\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)\\(1-n_{\varepsilon'})\delta(\varepsilon-\varepsilon'-\omega_q)\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)\\n_{\varepsilon'}\delta(\varepsilon+\varepsilon'-\omega_q)\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)\end{pmatrix}. \quad (6)$$

In the case of optical radiation when the frequency of the field ω_0 significantly exceeds the value of gap Δ , the source $Q(\varepsilon)$ produces nonequilibrium excitations in a wide energy region $\Delta\le\varepsilon\le\omega_0-\Delta$, and the inequalities

$$Q(\varepsilon)\ll\left(S^R,S^+,S^-\right) \quad (7)$$

are fulfilled at energies $\varepsilon \gg \Delta$ at arbitrary (obtainable in experiments) intensities of the electromagnetic field [14].

In contrast to the work [14], where the function $\alpha^2(\omega_q)F(\omega_q)$ was chosen to be $\lambda\,\omega_q^{k+1}/2\,\omega_D^{k+1}$ and the cases $k = 0, \pm 1$ were studied (λ is the dimensionless constant of electron-phonon interaction) we shall consider a "two-peak" approximation for this function:

$$\alpha^2(\omega_q)F(\omega_q) = a_1\delta(\omega_q - \omega_1) + a_2\delta(\omega_q - \omega_2)\,, \tag{8}$$

where a_1 and a_2 are some constants. We shall assume that the value of Δ is known and the inequality

$$\omega_1 \ll \Delta \ll \omega_2 \tag{9}$$

is satisfied. Using the Eq. (5) we can write with the help of inequality (7):

$$n_\varepsilon = \frac{Q(\varepsilon) + S^+(\varepsilon)}{S^R(\varepsilon) + S^+(\varepsilon) + S^-(\varepsilon)}\,. \tag{10}$$

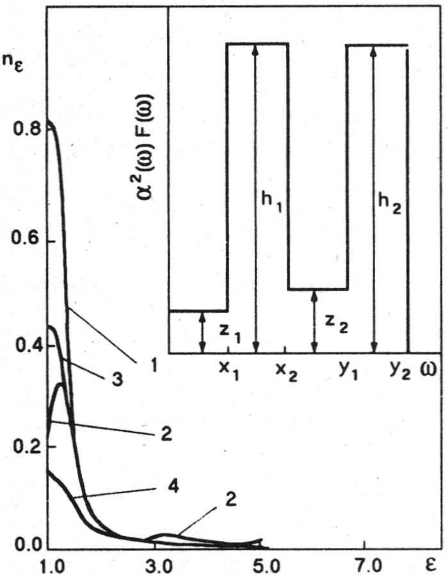
Substituting Exp. (8) into (6) and taking into account for the inequality (9), we can obtain from Eq. (10) at $\varepsilon \geq \Delta$:

$$n_\varepsilon \approx \left\{ \frac{(1 - n_{\varepsilon-\omega_1})[\,\varepsilon(\varepsilon - \omega_1) - \Delta^2]\sqrt{(\varepsilon + \omega_1)^2 - \Delta^2}\;\Theta(\varepsilon - \omega_1 - \Delta)}{n_{\varepsilon+\omega_1}[\,\varepsilon(\varepsilon + \omega_1) - \Delta^2]\sqrt{(\varepsilon - \omega_1)^2 - \Delta^2}} + 1 \right\}^{-1} \tag{11}$$

The analysis of this expression shows that $n_\varepsilon \approx 1$ in a narrow range of energies: $\Delta \leq \varepsilon \leq \Delta + \omega_1$ and is small at greater energies. Physical reasons for this result were discussed in [16]. We stress here that our simple approximation (8), which allowed us to carry out analytical calculations, is rather typical of some well-known superconductors, e.g., transition metals and alloys [17] and also high-temperature superconductors [18,19]. At the same time the numerical analysis of Eq. (2) using more proper expressions for $\alpha^2(\omega_q)F(\omega_q)$ is much more informative.

Numerical calculations. It is reasonable to begin the numerical analysis with a model approximation for the function $\alpha^2(\omega_q)F(\omega_q)$, depicted in the insert to Fig. 1 and to change then the parameters x_1, x_2, y_1, y_2, h_1, h_2, z_1, and z_2. As above, we shall treat the case of zero temperatures $T = 0$ ($N_{\omega_q} = 0$), and assume for a moment that the phonon field is longitudinal.

Let us consider curve 1 shown in Fig. 1. The corresponding spectral function is an analogue of the "two-peak" approximation treated above analytically. A characteristic feature here is that the boundary frequency of the low-energy phonon group is lower than the energy 2Δ of Cooper pairs. Changing the limiting values x_i and y_i one can obtain that the most radical variation (decrease) of the nonequilibrium function n_ε is taking place when the value x_2 exceeds the limit 2Δ (see curve 2 in Fig. 1) — this is fairly the threshold effect, caused by an additional "switching on"



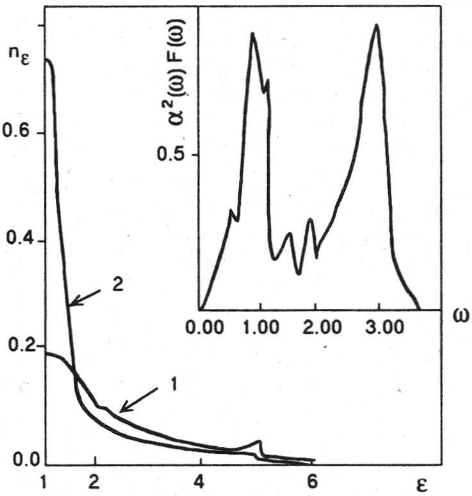


Fig. 1. Nonequilibrium distribution function of electron excitations n_ε at electromagnetic pumping with frequency $\omega_0 = 6$, parameter $\alpha_0 = 0.005$ and model spectral function $\alpha^2(\omega_q)F(\omega_q)$ (insert) with parameters: *1* — $x_1 = 0.05$; $x_2 = 1.9$; $y_1 = 3.2$; $y_2 = 4$; $h_1 = h_2 = 1$; $z_1 = z_2 = 0$; *2* — $x_2 = 2.3$; other parameters coincide with the curve *1*; *3* — $z_2 = 0.1$ and the others coincide with the curve *1*; *4* — $z_1 = z_2 = h_1 = h_2 = 1$. All the energy parameters are in Δ units

Fig. 2. Distribution function n_ε, computed by the spectral function $\alpha^2(\omega_q)F(\omega_q)$ taken from the experiment (insert): 1 — longitudinal phonon field; 2 — transverse one. All the energy parameters are in Δ units

of intermediate energy phonons with $\omega_q \geq 2\Delta$ into kinetic processes. The same effect may be observed immediately changing the value of z_2 (see curve 3 in Fig. 1). When z_2 increases up to the value 0.1 the inverse population disappears. Despite this, no threshold effect is observed, when the bounds of high-temperature phonon group are changed, but the latter also may influence significantly the form of function n_ε. For the sake of brevity we don't present here appropriate curves, but it must be noted, that at the displacement of the bound y_1 to greater values of ω_q the nonequilibrium function n_ε grows. Concerning parameters h_1 and h_2, it may be said that the function n_ε rises up to 1 (being obviously restricted by this value) when the ratio h_1/h_2 is increased. The role of extremely low energy phonons (described by the parameter z_1) is the most negligible. We shall stress also, that in the case of small values of α_0 the shape of the function n_ε weakly depends on these values. By the way, it must be noted that putting $z_1 = z_2 = h_1 = h_2 = 1$ we are coming to the situation, considered in [14] (the case of $k = -1$) with the same result which is interesting to compare with the novel ones (curve 4 in Fig. 1).

Discussion. A relative arrangement of phonon group boundaries and of value 2Δ is not a fixed one for any real superconductor and may be changed under the external influences, such as the magnetic field or temperature (due to the dependence of $\Delta(H,T)$). A typical experimental curve for $\alpha^2(\omega_q)F(\omega_q)$ (corresponding to 1-2-3 HTSC at $T=0$, $H=0$, see [18,19] is shown in the insert to Fig. 2. Calculations based on this curve gave the results shown in Fig. 2 (curve 1 — for longitudinal phonons, curve 2 — transverse ones). As can be noted from these curves, the change of sign in coherent factors $\left(1 \pm \frac{\Delta^2}{\varepsilon\varepsilon'}\right)$ of Eq. (2) plays a major role.

Here a question arises concerning the possibility of interaction of charge carriers with transverse phonons in superconductors. Such interaction may in fact take place in superconductors due to anisotropy of the crystalline lattice, the umklapp processes and in some other cases (see,e.g. [20]). To analyze the problem more properly both the partial functions $\alpha_\xi^2(\omega_q)F_\xi(\omega_q)$ for all the phonon modes ξ and the polarizations of these modes are to be used. Also it is necessary to know the value of 2Δ to normalize the phonon frequency scale. For such procedure used above, we have adopted 2Δ corresponding to the values typical of 1-2-3 HTSC. This is shown by the curve in the insert in Fig. 2, which simultaneously presents the most optimal case of the model function, considered in Sec.4.

Thus, the present analysis have shown that the high-temperature superconductors are very promising for obtaining of states, which are far from thermal equilibrium. Of course, in the case of HTSC, besides the above mentioned additional information, concerning partial modes of interaction of field-carriers, a considerable analytical work is to be done, which may give further proper predictions, only then the mechanism of superconductivity will become more apparent.

1. *Galitskii V. M., Yelesin V. F., Kopaev Yu. V.*, Pis'ma Zh. Teor. Fiz. **18**, 50 (1973).
2. *Kirzhnitz D. A., Kopaev Yu. V.*, Pis'ma Zh. Teor. Fiz. **17**, 379 (1973).
3. *Baru V. G., Sukhanov A. A.*, Pis'ma Zh. Teor. Fiz. **11**, 209 (1975).
4. *Aronov A. G., Gurevich V. L.*, Zh. Teor. Fiz. **65**, 1111 (1973).
5. *Gulian A. M., Zharkov G. F.*, Zh. Teor. Fiz. **84**, 1817 (1983).
6. *Otschik P., Eshrig H., Lange F.*, J. Low Temp. Phys. **43**, 397 (1981).
7. *Gregory W. D., Leopold L., Repici D., Bostock J.*, Phys. Lett. **A29**, 13 (1969).
8. *Gulian A. M., Nersesian O. N., Sergoyan G. M.*, DAN SSSR **304**, 1347 (1989).
9. *Gulian A. M., Zharkov G. F.*, Superconductors in external fields Nauka, Moscow (1990)p. 180.
10. *Testardi L. R.*, Phys. Rev. **B4**, 2189 (1971).
11. *Yanson I. K., Balkanshin O. P., Krasnogorov A. Yu.*, Pis'ma Zh. Teor. Fiz. **23**, 458 (1976).
12. *Jaworski F., Parker W. H.*, Phys. Rev. **B20**, 945 (1979).
13. *Mitsen K. V.*, in : Nonequilibrium superconductivity, ed. V. L. Ginzburg, Nova Science, New-York (1988), p. 161.
14. *Yelesin V. F.*, Zh. Teor. Fiz. **64**, 1755 (1974).
15. *Eliashberg G. M.*, Zh. Teor. Fiz. **61**, 1254 (1971).
16. *Gulian A. M.*, Fiz. Nizk. Temp. (1992), to be published.
17. *Vonsovskii S. V., Izyumov Yu. A., Kurmaev E. Z.*, Superconductivity of transition metals, their alloys and combinations, Nauka, Moskow (1977).
18. *Pickett W. E.*, Rev. Mod. Phys. **61**, 433 (1989).
19. *Carbotte J. P.*, Rev. Mod. Phys. **62**, 1027 (1990).
20. *Schrieffer J. R.*, Theory of superconductivity, Benjamin, New-York (1964).