УДК 539.374

B.M.Efros

SIMULATION OF PLASTIC DEFORMATION AND FRACTURE OF METALLIC MATERIALS UNDER HIGH HYDROSTATIC PRESSURE

A.Galkin Donetsk Physico-Technical Institute, Ukrainian National Academy of Sciences, 340114, R.Luxemburg str., 72, Donetsk, Ukraine

Paper have been received by 21 January 1997

In the paper a mathematical model of multiple material fracture at simple loading based on the percolation is presented. On the basis of this model the porosity corresponding to formation of macrodiscontinuity is evaluated. A kinetic equation for porosity with an account of the formation and the curing of microdiscontinuities is proposed. Conditions for obtaining the state with unlimited deformability at forming of metals under external pressure are analyzed. The proposed mathematical model of metal fracture is explained by an example of hydroextrusion process. Namely, the relationships for the extrudate damage are obtained and the conditions for the opening axial cavity are discussed.

According to current ideas [1,2] the metal fracture at plastic deformation is a multistage and multiscale process. It is developed at various structural levels. The lower levels of a fracture provoke the higher-level ones, i.e. the secondary defects are nucleated from the coalescence of the primary ones, the tertiary defects being formed from the secondary ones, etc. The progress of defects at the same level is self-modelled up to a certain moment, i.e. as the damage degreis increased the geometry of the defect cascade is transformed according to the similarity principle (scaling law) [2]. Violation of self-modelling is due to formation of another level defects [2].

Effective methods for the study of the above processes can be taken from the percolation theory [3]. We present some concepts of this theory which are the basis for the below model of material fracture at the plastic deformation under pressure.

Let us assume that (to a certain amount) a large volume of space v is randomly filled up with substance. At $v<<1$ the substance forms small regions separated from one another. At addition of substance these regions start to coalesce thus originating the formations (clusters).The cluster sizes increase and at some critical value v_c a cluster is formed which spreads over the entire volume. This cluster is referred to as an infinite cluster. For three-dimensional regions, $v_c \approx 0.17$ [3]. The clusters are of completely random shape. But on the average, the geometry of randomly arranged substance has quite definite properties. One of them is reflected by the hypothesis of similarity according to which the cluster size distribution function related to a mean value at given v is unchanged and independent on v. The similarity is upset at the moment of the infinitive cluster formation.

Let us assume the following model of fracture. At each level there is a corresponding simple defect, i.e. the fracture atom. The bound set (cluster) of fracture atoms forms the this level. An infinite cluster of fracture atoms of one level gives a fracture atom of another level. Thus the defects are considered to correspond to the clusters. This results in the similarity principle for defects substantiated in [2] which states that at a continuous fracture the cascade of defects is transformed in similar manner, and all the linear sizes of cascade grow proportionally to a statistically average size of defects. Actually the similarity principle, in the given case, is a manifestation of regularity in the cluster development characterized by the similarity hypothesis of the percolation theory.

According to the similarity principle, the cluster geometry of the defect cascade is completely determined by the relative volume of defects. In [4], we obtain the kinetic equation for this quantity which is further referred to as porosity.

This equation is:

$$\frac{d\theta}{d\gamma} = \left[\alpha\sqrt{1+\frac{3}{2}\theta\left(\frac{\sigma}{\tau}\right)^2} + \frac{3}{2}\theta\frac{\sigma}{\tau}\right], \tag{1}$$

where Θ – porosity, γ – intensity of shear strain, σ – hydrostatic constituent of stress tensor, τ – intensity of tangential stress, α – internal friction coefficient.

This relation implies that both the formation of micropores and their curing (at $\sigma < 0$) take place at plastic deformation. The processes correspond to the first and the second terms in the right-hand side of relation (1) respectively.

From the equation (1) it follows that at $\frac{\sigma}{\tau} < 0$, the some equilibrium porosity (Θ_e) becomes steady in the material under deformation. The value of this porosity is a root of an equation:

$$\alpha\sqrt{1+\frac{3}{2}\Theta_e\left(\frac{\sigma}{\tau}\right)^2} + \frac{3}{2}\Theta_e\frac{\sigma}{\tau} = 0. \tag{2}$$

According to [5] the criterion of metal fracture at the pressure treatment is the appearance of a simple defect at the macrolevel. On the basis of the proposed model we evaluate the "critical" loosening due to which this effect appears in material.

According to [1] the formation of microdiscontinuity with the linear size of the order of 0.1 μm is an elementary act of fracture. The coalescence of these defects gives microdiscontinuities with the size of the order of the linear size of the structural heterogeneity (block, grain). The coalescence of the latter ones results in the appearance of a macrocrack. In terms of the proposed model we have: a microdiscontinuity of about 0.1 μm is the first-level fracture atom; a microdiscontinuity of the structural heterogeneity size is the second-level fracture atom; a macrodiscontinuity is the third-level fracture atom. According to the percolation theory the relative portion of this substance should reach its value v_c for the formation of an infinite cluster of any substance. Therefore, the nucleation of the third-level fracture atom per unit volume requires v_c of the second-level fracture atoms and $v_c \cdot v_c$ of the first-level ones. Thus the critical loosening is evaluated as $\Theta_c = v_c^2$. Substituting v_c = 0.17 into the expression, we obtain $\Theta_c \approx 3\%$ which corresponds by the order of magnitude to experimental value $\Theta_c \approx 1\%$ [1]. At $\Theta_c < \Theta_e$ the solid fracture is before its equilibrium value. And at $\Theta_c > \Theta_e$ the value Θ_e will be reached before the solid starts to fracture. The state with the unlimited deformability should correspond to the given condition.

plastic deformation of metals without fracture under the external pressure. The above model of metal fracture can be explained by an example of steel loosening at hydrostatic extrusion [6,7]. According to [1,8] for the steels; $\alpha \sim 10^{-2}$, $\Theta < 10^{-2}$. It follows from [5] that at the hydrostatic extrusion $\sigma/\tau \sim -1...-3$. In view of this fact and neglecting the terms of the second order in Eq. (1) one gets the simplified kinetic equation for porosity

$$d\Theta = \left(\alpha + \frac{2}{3}\Theta\frac{\sigma}{\tau}\right)d\gamma. \tag{3}$$

It follows from Ref.[9] that at small magnitude of the contact friction which is typical for the hydroextrusion, along the die axis

$$\frac{\sigma}{\tau} = -\sqrt{3}\ln\left(\frac{\rho}{\rho_2}\right)^2 - \frac{2}{3}, \tag{4}$$

$$d\gamma = -\frac{2\sqrt{3}}{\rho}d\rho, \tag{5}$$

where ρ is the radius of the arbitrary point along the die axis, ρ_1, ρ_2 is the radius of the deformation area boundaries.

Substituting relationships (4), (5) into Eq. (3) one gets the following differential equation for deformation of Θ:

$$\frac{d\Theta}{dx} - 18\Theta x - 6\Theta = -2\sqrt{3}\alpha, \tag{6}$$

where $x = \ln(\rho/\rho_2)$.

The initial condition: $\Theta = \Theta_1$ at $x = x_1$, where Θ_1 is the porosity of the initial billet, $x_1 = \ln(\rho_1/\rho_2)$.

The solution of Eq.(6) at this initial condiction has the form

$$\Theta(x) = e^{-F(x)}\left[\Theta_1 - 2\sqrt{3}\alpha\int_{x_1}^{x} e^{F(x)}dx\right], \tag{7}$$

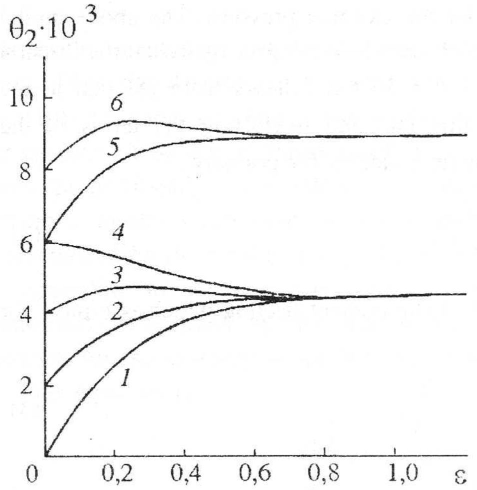
where $F(x) = 3(x_1 - x)[2 + 3(x + x_1)]$. Substituting $x = 0$ into Eq. (7) one gets the expression for the porosity of the extrudate Θ_2:

$$\Theta_2 = \Theta_1 e^{-3x_1(2+3x_1)} + 2\sqrt{3}\alpha\int_0^{x_1} e^{-3x(2+3x)}dx.$$

Replacing the variables in the integral one can reduce the latter to the probability integral for which there are the tables [10]. Having performed simple transforms one gets:

$$\Theta_2 = \Theta_1\exp\left[-\frac{2}{3}\varepsilon\left(2+\frac{3}{2}\varepsilon\right)\right] + 2\alpha e\sqrt{\frac{\pi}{3}}\cdot\left\{\Phi\left[\sqrt{2}\left(1+\frac{3}{2}\varepsilon\right)\right] - \Phi\left(\sqrt{2}\right)\right\}, \tag{8}$$

where $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{1}{2}\xi^2}d\xi$ is the normal distribution function [10], $\varepsilon = \ln(\rho_1/\rho_2)^2$ is the logarithmic deformation at the hydrostatic extrusion. Fig. 1 presents the plots of $\Theta_2 = \Theta_2(\varepsilon)$ at various values of α and Θ_1.



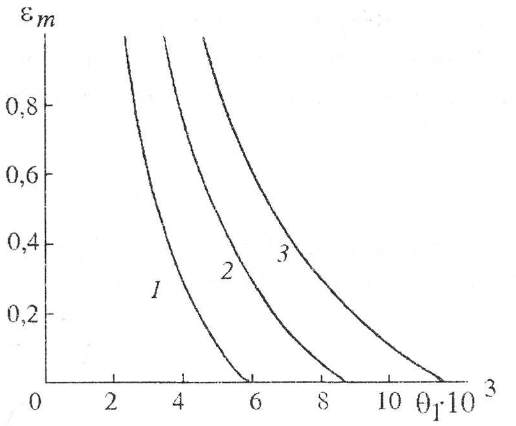


Fig. 1. Extrudate porosity as a function of logarithmic deformation Θ_2 at hydrostatic extrusion ε : *1* – $\Theta_1 = 0$, $\alpha = 10^{-2}$; *2* – $\Theta_1 = 2\cdot10^{-3}$, $\alpha = 10^{-2}$; *3* – $\Theta_1 = 4\cdot10^{-3}$, $\alpha = 10^{-2}$; *4* – $\Theta_1 = 6\cdot10^{-3}$, $\alpha = 10^{-2}$; *5* – $\Theta_1 = 6\cdot10^{-3}$, $\alpha = 2\cdot10^{-2}$; *6* – $\Theta_1 = 8\cdot10^{-3}$, $\alpha = 2\cdot10^{-2}$

Fig. 2. Deformation producing maximum loosening of metal ε_m as a function of the billet porosity Θ_1 : *1* – $\alpha = 10^{-2}$; *2* – $\alpha = 1.5\cdot10^{-2}$; *3* – $\alpha = 2.0\cdot10^{-2}$

It is clear from this figure that the hydroextrusion shows three versions of dependence of the extrudate porosity on the deformation degree: porosity growth with the increase in the deformation degree; decrease in porosity with the deformation degree growth; dependence at the porosity maximum.

All above cases are implemented in practice. The first one is typical of the hydroextrusion of metals with small initial damage [11], the second case is observed at the hydroextrusion of billets with a great amount of microdiscontinuities [7]. The third case is realized at the hydroextrusion of tool steels when at small and great deformations the extrudate quality is high whereas at average deformations plasticity drops down to the axial destruction of the extrudate [11].

Now let us determine the deformation degree at which the extrudate porosity is the maximum one. For this purpose we find the derivative $d\Theta/d\varepsilon$. Eq. (8) yields

$$\frac{d\Theta_2}{d\varepsilon} = \exp\left(1 - t^2\right)\left(\alpha\sqrt{3} - 3\Theta_1 t\right), \tag{9}$$

where $t = 1 + \frac{3}{2}\varepsilon$. At $\varepsilon = \varepsilon_m$ where

$$\varepsilon_m = \frac{2}{3}\left(\frac{\alpha}{\sqrt{3}\Theta_1} - 1\right), \tag{10}$$

the number of defects in the extrudate is maximum. Fig.2 presents the plot $\varepsilon_m = \varepsilon_m(\Theta)$ at various values of α.

The proposed mathematical model allows one to write the condition at which the hydroextrusion will result the formation of axial cracks in the extrudate. Thus according to Eq. (8) this condition has the form:

$$\Theta_1 \exp\left[-\frac{3}{2}\varepsilon\left(2 + \frac{3}{2}\varepsilon\right)\right] + 2\alpha e\sqrt{\frac{\pi}{3}}\cdot\left\{\Phi\left[\sqrt{2}\left(1 + \frac{3}{2}\varepsilon\right)\right] - \Phi\left(\sqrt{2}\right)\right\} < \Theta_c, \tag{11}$$

Let us consider the case of $\Theta_1 = 0$. Thus Eq. (11) yields the following equation to determination of the critical deformation ε_c.

$$\Phi\left[\sqrt{2}\left(1+\frac{3}{2}\varepsilon_c\right)\right]-\Phi\left(\sqrt{2}\right)=\frac{\Theta_c}{2\alpha e}\sqrt{\frac{3}{\pi}} \tag{12}$$

Fig.3 shows the curve of the dependence of the left-hand side of Eq.(12) on the magnitude ε. Using this curve one can easily find the magnitudes ε_c. Thus at $\alpha = 0.02$ and $\Theta_c = 8\cdot10^{-3}$ the right-hand side of Eq.(12) is equal to $7.2\cdot10^{-2}$ and the critical deformation $\varepsilon_c \approx 0.5$.

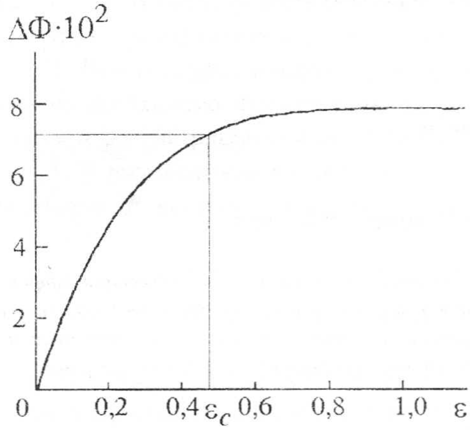


Fig. 3. Dependence of the left-hand side of Eq.(12) $\Delta\Phi$ on the magnitude ε.

The proposed model of metal failure is illustrated by an example of the hydroextrusion process. In this case the model makes possible to explain the reduction in the extrudate plasticity at the average deformation degree and to determine the most unfavorable deformation (ε_m) from this point of view. On the basis of this model the condition for hydroextrusion without the axial fracture of the extrudate is obtained (Eq.(11)).

1. *V.I.Betekhtin, V.I.Vladimirov, A.G.Kadomtsev, A.I.Petrov*, Problemy prochnosti № 7, 38 (1979).
2. *G.L.Barenblatt, L.P.Botvina*, Fiziko khimicheskaja mekhanika materialov **22**, №1, 57 (1986).
3. *V.I.Shklovskii, A.L.Efros*, Electronic properties of doped semiconductors, Nauka, Moscow (1979) (in Russian).
4. *Ya.Beigelzimer,L.Loladze,B.Efros*, High Pressure Research **13**, 171 (1994).
5. *V.L.Kolmogorov*, Stress, Strain, Failure, Metallurgija, Moscow (1970)(in Russian).
6. *B.Avitzur*, Metal forming:Processes and Analysis, McGraw–Hill, NewYork (1968).
7. *B.I.Beresnev, K.I.Ezerskii, E.V.Trushin, B.I.Kamenetskii*, High Pressure in Modern Technologies of Material Treatment, Nauka, Moscow (1988) (in Russian).
8. *V.V.Novozhilov*, Prikladnaja matematika i mekhanica **29**, №4, 680 (1965).
9. *V.V.Sokolovskii*, Theory of Plasticity, Vysschaja Shkola, Moscow (1969) (in Russian).
10. *E.S.Ventzel*, Probability Theory, Nauka, Moscow (1969) (in Russian).
11. *Yu.F.Cherny, V.Z.Spuskanjuk, A.A.Ljadskaja, A.I,Opanashuk*, Hydropressing of Tool Steels, Tekhnika, Kiev (1987) (in Russian).