PACS: 72.55.+s, 72.15.Nj

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COUPLING OF ELECTROMAGNETIC MODES AND ANOMALOUS SKIN-EFFECT IN MAGNETIC FIELD

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Received 22 January, 1999

Based on the analysis of the dispersion relation for the low-frequency collective modes in the metals electron plasma, a nontraditional point of view on the nature of the anomalous skin-effect in the presence of a static magnetic field is proposed. It is shown that the last is the result of specific coupling of the electromagnetic modes, corresponding to the roots of the dispersion equation, and depends on the concrete electron energetic spectra's shape.

1. A concept of the anomalous skin-effect [1,2] is the classic one. Its theory has been elaborated in detail [3–5]. As a rule, a complicated boundary problem is solved, and field distribution inside the metal and metal's surface impedance are calculated. But this paper does not deal with these aspects of the problem and concerns only with the analysis of the dispersion equation and spectra of the collective electromagnetic modes in metal. In the situation studied here, a simple, but realistic model of noncompensated metal is considered. Nevertheless, the general character of our consideration is not limited. As a result, a nontraditional point of view on the nature of the anomalous skin-effect in the presence of a static magnetic field H has been proposed. Our findings may be summarized briefly as follows. At high magnetic field, if the conductivity depends on the wave number, the skin-effect is a result of the coupling of collective excitations. In the case considered here, these are helicon [6–9] («correct» mode), doppleron [10–13] («wrong» mode) and damped helicon.

2. Let us consider the spectrum of the electromagnetic modes in noncompensated metal in the range of frequencies and fields limited by inequality $\omega \ll \nu \ll \omega_c$ for the case of a Fermi surface (FS) with axial symmetry about the direction of $\mathbf{k} \parallel \mathbf{H} \parallel \mathbf{z}$. Here $\mathbf{k}$ is the wave vector; ω is the wave frequency; ν is the collision frequency; $\omega_c = eH/mc$ is the cyclotron frequency. The dispersion equation for the circularly polarized components of the electric field $E_\pm = E_x \pm$ $\pm iE_y$ is

$$k^2c^2 = 4\pi i\omega\sigma_\pm(k), \text{ (}\pm\text{ polarization)}, \quad (1)$$

where $\sigma_\pm(k) = \sigma_{xx} \pm i\sigma_{yx}$ is the conductivity.

Let us approximate the noncompensated FS by electron surface being given by [8] («corrugated cylinder»):

$$S(p_z) = S_0 + S_1 \cos(\pi p_z/p_0), \ |p_z| \le p_0, \quad (2)$$

where p_z is the component of electron momentum parallel to the $\mathbf{H}$; $S(p_z)$ is the cross-section area of the FS on a plane of constant p_z; S_0, $S_1 < S_0$ and p_0 are the model parameters. Here and later on we imply that m and v are independent of p_z.

The model (2) exhibits realistic properties: (i) there are the cross-sections with extreme $S(p_z)$ ($\partial S/\partial p_z = 0$) values; (ii) there are the cross-sections with extreme $\partial S/\partial p_z$ values. So, the $|\partial S/\partial p_z|$ value continuously varies from zero to the maximum value.

It is convenient to introduce dimensionless parameters:

$$q = kv_m/\omega_c,\ \xi = \omega_c^3 c^2/\omega_p^2 \omega v_m^2,\ \gamma = (\nu - i\omega)/\omega_c, \quad (3)$$

where $v_m = |\partial S/\partial p_z|_{max}/2\pi m$ is the maximum electron velosity along **H**; $\omega_p = (4\pi Ne^2/m)^{1/2}$ is the plasma frequency; N is the electron concentration. Using the known expression for the conductivity (see e.g. [8,9]) we rewrite the dispersion equation in the form

$$\mp\, q^2\xi^3 = F_\pm(q), \quad (4)$$

$$F_\pm(q) = [(1 \pm i\gamma) - q^2]^{-1/2}. \quad (5)$$

Here $F_\pm(q)$ is the nonlocal multiplier at the conductivity: $\sigma_\pm = \pm\, i(Nec/H)F_\pm$.

It is most easily to see the nature of the roots of equation (4) by solving it graphically in the collisionless limit ($\gamma \to 0$). This is done in the insert of Fig. 1, $F_0 = F_\pm|_{\gamma\to 0}$. In the local ($q \to 0$) limit $F_0 = 1$ and (4) has a real root G_L (helicon) for «–» polarization and an imaginary root G'_L (damped helicon) for «+» polarization for any values of ξ. Spectra of these modes are shown in Fig.1: $q^2_{G_L} = \xi^{-3}$; $q^2_{G'_L} = -\xi^{-3}$ (curve L).

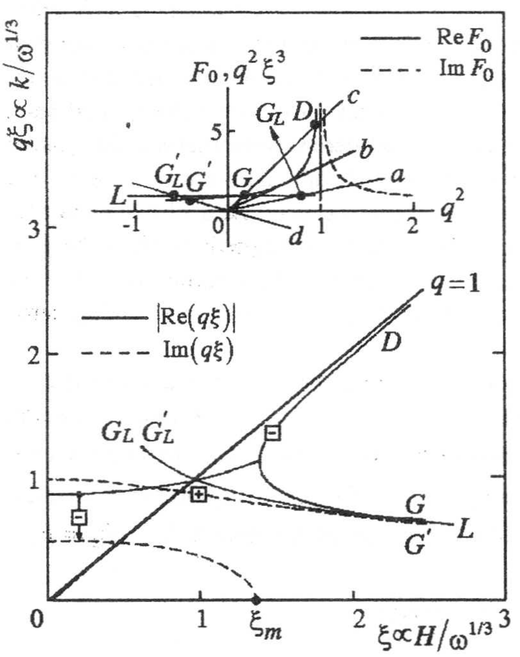

Fig. 1. The spectra of electromagnetic modes in noncompensated metal with conductivity (5) (the roots of Eq. (4), $F_0 = F_\pm|_{\gamma = 0}$); G, G' and D are the helicon, damped helicon and doppleron solutions, respectively. G_L and G'_L are helicon and damped helicon in the local limit. The polarity of circular polarization is indicated in squares; $\xi_m^3 = (27/4)^{1/2}$. Insert: graphical solution of (4), (5). The curves a, b and c are the left-hand side for «–» polarization for progressively larger values ξ. The curve d is the left-hand side for «+» polarization. The curve L is the function F_0 in the local ($q \to 0$) limit

The non-local effects result in the following important consequences:

(i) At $q^2 \to 1_{-0}$ the value of Re $F_0 \to \infty$. As a result, for «–» polarization there may be either no real root (curve a) or two roots (curve c): G (helicon) and D (doppleron). The critical value of ξ, $\xi = \xi_m = (27/4)^{1/6}$ (helicon edge), separates these regions (curve b).

(ii) At $q^2 > 1$ the value F_0 becomes imaginary. A complex value F_0 indicates that the wave is severely damped. Hence, $q^2 = 1$ defines the position of a damping edge (Kjeldaas edge [10]).

The mechanism responsible for damping edge is well known as the Doppler-shifted cyclotron resonance (DSCR). In the case considered here, the existence of the damping edge is caused by the above-mentioned property of surface (2): $0 \le |\partial S/\partial p_z| \le |\partial S/\partial p_z|_{max}$.

3. It is impossible to obtain complex solutions of (4) at $\xi < \xi_m$ for «–» polarization by the graphical method. However, we can easily obtain exact analytic solutions of the equations (4). The full spectrum of the electromagnetic modes corresponding to these solutions is shown in Fig. 1. Here $q\xi \sim k/\omega^{1/3}$, $\xi \sim H/\omega^{1/3}$ are universal coordinates [11].

Fig. 1 shows the spectra of lightly damped waves propagating only along the positive direction of axis $\mathbf{z}$ ($\mathrm{Im}(q\xi) > 0$ and group velosity $v_{gr} = d\omega/dk > 0$). In collisionless limit ($\gamma \to 0$) from (4) one obtains

$$v_{gr} = 2v_{ph}(\xi^3 - \partial F_0/\partial(q^2))/\xi^3. \tag{6}$$

A glance at Fig. 1 (insert) shows that for doppleron $\xi^3 < \partial F_0/\partial(q^2)$ ($v_{gr} > 0$, $v_{ph} < 0$; «wrong» mode) whereas for helicon $\xi^3 > \partial F_0/\partial(q^2)$ ($v_{gr} > 0$, $v_{ph} > 0$; «correct» mode). Therefore, at $\xi < \xi_m$ Fig. 1 gives not one but two solutions of (4) for «–» polarization corresponding to the modes with equal, but oppositely directed phase velocities. For «+» polarization the solution is purely imaginary. For high q^2 in the essentially nonlocal limit ($q^2 >> 1$) we have $F_\pm \cong i/q$ and equation (4) becomes $\mp q\xi^3 \cong i$ ($k^3 \cong \mp i\omega_p^2\omega/c^2v_m$). This equation has three solutions, which are valid for relatively low values of ξ (see Fig. 1). Solutions for «–» polarization coincide with the known solutions corresponding to the «classic» anomalous skin-effect regime. Hence, we have reasons to suppose that at $\xi < \xi_m$ Fig. 1 describes the anomalous skin-effect in the magnetic field presence.

4. The used model of FS allows one to analyse the obtained spectra from the nontraditional point of view. This is in the first place connected with the simplicity of function (5). We see that equations (4), (5) may be written as

$$\left(q^2 - q_{GL}^2\right)\left(q^2 - q_{G'L}^2\right)\left(q^2 - q_D^2\right) = -q^2/\xi^6, \tag{7}$$

where $q_D^2 = 1$.

It is seen that the dispersion equation can be interpreted as the equation of three coupled modes. Because q_{GL}^2 and $q_{G'L}^2$ are solutions of (4) in the local limit (curve L in Fig. 1), we have the reasons to identify two of the modes as helicon and damped helicon. The third solution of (7) is a straight line $q^2 = 1$, which corresponds to the damping edge of waves due to the DSCR (Fig. 1). So, corresponding mode can be called the DSCR-mode or doppleron. Fig. 1 shows that at relatively high values of ξ, far from the point of the modes degeneracy ($\xi = 1$) the coupling can be neglected. With ξ decrease the coupling constant q^2/ξ^6 in the righthand side of (7) is growing, and G and D branches of the spectra are bending, becoming hybridized in the vicinity of ξ_m. As a result, in the interval $\xi < \xi_m$ «a gap» arises and solutions of (7) for «–» polarization in this region become complex. It should be noted that the both modes have positive phase velocity whereas doppleron has negative group velosity («wrong» mode). They propagate one towards another. Namely this circumstanse leads to the binding of respective spectral branches and formation of the «gap». In the opposite case the dispersion curves of both modes split rather than bind and the «gap» does not arise. Helicon-phonon coupling (both modes are «correct») illustrates this point well enough [12].

As was shown above, all three solutions of the dispersion equation describe the anomalous skin-effect in the presence of the external magnetic field. Hence, the anomalous skin-effect can be treated here as the result of coupling of helicon, damped helicon and doppleron modes. Unlike the case of the helicon-phonon coupling, all these modes are collective excitations of one and the same subsystem of metal, i.e. the electron plasma of metal. However, there is the mechanism of their coupling. These are the nonlocal effects resulting, first, in the appearance of a doppleron solutions of the dispersion equation and, second, in the hybridization of spectral branches.

5. Above (see 2. (ii)) it was shown that the existence of the anomalous skin-effect is due to the collisionless damping (at $q^2 > 1$ function F_0 is imaginary). However presence of the damping is not so essential as it seems at a first glance. Let us consider the FS model: $S = S_0(1 - |p_z/p_0|)$, $|p_z| \le p_0$ («parabolic lens» [13]). The function F_0 ($\gamma \to 0$) for this model is given by

$$F_0 = (1 - q^2)^{-1}. \quad (8)$$

Graphical solution of (4), (8) is shown in the insert of Fig. 2. The function F_0 is purely real for all q^2 which evidences the absence of collisionless damping of waves. Indeed, for this FS model the DSCR condition is fulfilled only at $q^2 = 1$. Newertheless, the helicon (G) and doppleron (D) branches for «–» polarization bind together and at $\xi < \xi_m$, $\xi_m = 4^{1/3}$, a «gap» arises. As a result, the solutions of the dispersion equation become complex (Fig. 2). It is evident that these solutions as well as solution G' for «+» polarization describe the skin-effect. This is not the classical anomalous skin-effect (compare Fig. 1 and 2), but it is important that the skin-effect exists, though the damping edge and the region of collisionless damping, continuous along the q^2-axis, are absent.

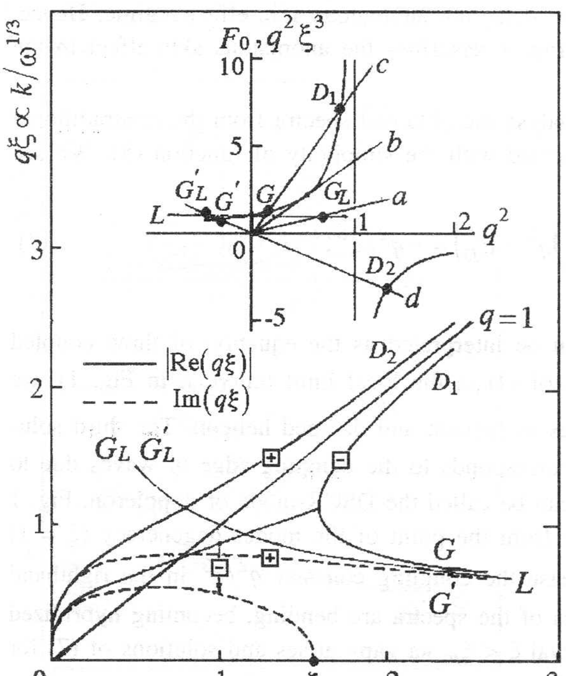

Fig. 2. The same as in Fig. 1, but for noncompensated metal with conductivity (8); D_1 and D_2 – the doppleron solutions for «–» and «+» polarizations, respectively; $\xi_m^3 = 4$

6. So, in this paper, we have analysed the nature of the anomalous skin-effect in metals in the external magnetic field. From the classical point of view the anomalous skin-effect is due to nonlocal effects and existence of the region of collisionless damping of the waves. The function $F_0 \cong i/q$ when $q^2 >> 1$, and for low maghetic fields (ω = const) the roots of the dispersion equation are almost independent of H (Fig. 1). We suggest that the anomalous skin- effect can be interpreted as a result of hybridization of coupled electromagnetic modes. It is important that

one of these modes, the doppleron, has positive group velocity and negative phase velocity («wrong» mode). Indeed, using (4), we find

$$\partial|q\xi|/\partial\xi = \frac{|q|}{2}\frac{\xi^3 + 2\partial F_0/\partial(q)^2}{\partial F_0/\partial(q)^2 - \xi^3}. \qquad (9)$$

Now from (9), it is seen that for helicon $\partial|q\xi|/\partial\xi < 0$ ($v_{gr} > 0$, $v_{ph} > 0$), whereas for doppleron $\partial|q\xi|/\partial\xi > 0$ ($v_{gr} > 0$, $v_{ph} < 0$) and $\partial|q\xi|/\partial\xi|_G^D \to \pm\infty$ when $\xi \to \xi_m$. The mechanism responsible for the coupling is the same, i.e. nonlocal effects.

The proposed point of view is confirmed by the analysis of the electromagnetic modes spectra in metal with conductivity (8). As is mentioned above, for this model the DSCR condition is fulfilled only at $q^2 = 1$ and collisionless damping is absent at $q^2 > 1$. However, in this case the nonlocal effects also produce the regime of coupling modes and for $\xi < \xi_m$ there are three roots of the dispersion equation describing the anomalous skin-effect. But, because the function $F_0 = -1/q^2$ when $q^2 >> 1$, the roots of (4), (8) for low magnetic fields (ω = const) are strongly H-dependent (Fig. 2).

The «parabolic lens» is unsufficiently realistic model, as for this model the value of $|\partial S/\partial p_z|$ is independent of p_z. Real FS has extreme cross-sections $S(p_z) = S_m(\partial S/\partial p_z = 0)$. Now expand $S(p_z)$ about p_m, a point of extremum $S(p_z)$. For small $(p_z - p_m)$ the cross-section can be written as $S(p_z) \approx a + b|p_z - p_m|^n$, where $n > 1$. Using expression for the conductivity we can easily show that the «skin» roots H-dependence is model dependent. The anomalous skin- effect disappears for low magnetic fields in the limit $n \to \infty$ (cylindrical FS). When $n \to 1$, we have «parabolic lens» model. The classical anomalous skin-effect regime arises for the special case $n = 2$. See e.g. the model (2) and free-electron model.

The work has been supported by the Foundation of Fundamental Research of Ukraine, Grant N 2.4/211.

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