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A MODEL OF THE FRACTAL STRING

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A theoretical description of the fractal string model using the mathematical formalism of the fractional calculation is given. The fractal properties of such model are investigated. The anomalous behavior of the plastic subsystems of fractal string is analyzed on the basis of the given model.

1. Introduction

Lately the scientists actively try to explain different anomalous properties of physical object [1] on the basis of the definition of the fractal string (first introduced in Geometry [2]). At investigating the meso- and microscopic nonuniform real structure of magnetite [3] clusters have been discovered which can be classified as fractal objects. In high-temperature superconductors in the region of superconducting transition [4] on the lines of the temperature absorption spectra (in the maxima) there were "shifts"of different width and depth where the magnetic susceptibility behaves stochastically. There are anomalies in the form of singularities such as plateau and tightened "tails" (fractal properties) on the temperature dependencies of the absorption coefficient, velocity of sound pulses propagation [5], linewidth of the nuclear magnetic resonance [6] near the orientational phase transitions in orthoferrites. Adequate descriptions of the anomalous behavior of physical parameter near phase transitions in real samples of magnets, ferroelectrics, high-temperature superconductors require the model of the nonlinear lattice with the spontaneous deformation to be further developed on the basis of qualitatively new representations on the nature of fractal and stochastic properties of the lattice. For the theoretical descriptions of fractal objects it was proposed [7,8] to use the theory of fractional calculations [9]. In [7] the connection between Cantor fractal quantity and fractional integral has been established, that makes it possible to propose the evolution equations with fractional time derivatives for a series of model physical systems. In [8] equations with fractional space-time derivatives have been introduced to describe the kinetics of particles and anomalous transport. The aim of this paper is to introduce equations with fractional space-time derivatives for the description of fractal string model dynamics.

2. The basic equations of the fractal string

Let x be the coordinate of a point on a fractal string in a moment of time t . In a process of evolution the point of the string turns from the state (x',t') to the state (x,t). In this case displacement u' depends on states (x,t) and (x',t') in a non-local way. It is necessary to express u' by means of u, depending only on current state (x,t). This can be achieved with the help of expansion of u' in the fractional series of Taylor under the conservation of the first elements of expansion

$$u' = c_2 D_1 u + c_3 F_1 u; \quad D_1 = D_t^{\nu}; \quad D_t^{\nu} u = \frac{1}{\Gamma(1-\nu)} \partial_t \int_{t'}^{t} u(x,\tau)|t-\tau|^{-\nu} d\tau;$$
$$c_2 = |t-t'|^{\nu} / \Gamma(1+\nu); \quad c_3 = |x-x'|^{\alpha} / \Gamma(1+\alpha); \quad F_1 = D_x^{\alpha}, \tag{1}$$

where ν,α are indices of order $(0 \le \nu \le 1;\ 0 \le \alpha \le 1)$ of fractional partial Riemann-Liouville D_t^{ν}, D_x^{α} derivatives operators (left-side, if $t > t', x > x'$; right-side, if $t < t', x < x'$) according to variables t, x; Γ is gamma-function; ∂_t, ∂_x are operators of usual partial derivatives. Using (1), for a speed v' we get

$$v' = \partial_t u' = v_1 + v_2 + v_3; \quad v_k = c_k p_k; \quad p_k = D_k u; \quad k = 1, 2, 3;$$
$$c_1 = \nu |t-t'|^{\nu-1} sign(t-t') / \Gamma(1+\nu); \quad D_2 = D_t^{1+\nu}; \quad D_3 = D_x^{\alpha} \partial_t, \tag{2}$$

where v_1, v_2, v_3 are speeds of plastic, elastic and creep shifts, respectively, of fractal string. For distortion β' we find

$$\beta' = \partial_x u' = \beta_1 + \beta_2 + \beta_3; \quad \beta_k = g_k q_k; \quad q_k = F_k u; \quad F_2 = D_x^{1+\alpha};$$
$$g_1 = \alpha |x-x'|^{\alpha-1} sign(x-x') / \Gamma(1+\alpha); \quad g_2 = c_3; \quad g_3 = c_2; \quad F_3 = D_t^{\nu} \partial_x, \tag{3}$$

where $\beta_1, \beta_2, \beta_3$ are plastic, elastic and creeper distortions of fractal string. It follows from (2) that for v_1 the anomalous dependencies on t near t' close to a type λ-point or an occurrence of "jump" because of $sign(t-t')$ multiplier availability are typical. Similar anomalous dependencies on x close to x' for β_1 follow from (3). For v_2, v_3, β_2, β_3 anomalies of "a soft mode " type (at $t = t'$ or $x = x'$ are zero) are typical. Then we get for energy density W of fractal string and receive action J

$$W = W_1 + W_2; \quad 2W_1 = \rho_{kn} v_k v_n = a_{kn} p_k p_n; \quad J = \iint (W_1 - W_2) dx dt;$$
$$2W_2 = \kappa_{kn} \beta_k \beta_n = b_{kn} q_k q_n, \tag{4}$$

where W_1, W_2 – density of kinetic and potential energy; ρ_{kn}, κ_{kn} – tensors of density and force parameter of string. Non-diagonal components of tensors describe relations between plastic, elastic and creep subsystems of fractal string. An action J is a functional of p_k, q_k, that's why the necessity of fractional variation calculation appears. We have attempted to solve the problem for the functionals of a specified type and have got the required equations

$$D_k\left(a_{kn}D_n u\right) = F_k\left(b_{kn}F_n u\right), \tag{5}$$

where summation with respect to index n is meant. Under $\nu=\alpha=0$ from (5) a wave equation for an ordinary string follows. A particular type of a combination of the operators with the right or left - hand derivatives in pairs D_k, D_n or F_k, F_n in (5) depends on choice of boundary conditions.

3. On commutation relations and semigroup properties of fractional derivatives operators

In the theory of the fractional calculation the semigroup property for the fractional integral operators I^{α}_{a+} is known [9] to be

$$I^{\alpha}_{a+} I^{\beta}_{a+} f(x) = I^{\beta}_{a+} I^{\alpha}_{a+} f(x) = I^{\alpha+\beta}_{a+} f(x); \quad \alpha>0,\ \beta>0 \tag{6}$$

where $f(x)$ pertain to functions, which can be integrated, according to Lebege, of finite segment $[a, b]$, i.e. $f(x) \in L_1\ (a, b)$. According to definition for the left-side fractional integral we note

$$I^{\alpha}_{a+} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-\xi)^{\alpha-1} f(\xi)d\xi. \tag{7}$$

From (6) it follows that the fractional integral operator with different ($\alpha \neq \beta$) or equal ($\alpha = \beta$) indices of order are commuted, i.e. commutator $\left[I^{\alpha}_{a+}, I^{\beta}_{a+}\right] f(x) = \left(I^{\alpha}_{a+} I^{\beta}_{a+} - I^{\beta}_{a+} I^{\alpha}_{a+}\right) f(x) = 0$. The semigroup property (6) is fulfilled also at $\beta < 0$, $\alpha > 0$, if $f(x)$ pertain to functions which can appear as fractional integral, i.e. $f(x) \in I^{-\beta}_{a+}(L_1)$. If the pointed condition is disturbed, i.e. $f(x) \notin I^{-\beta}_{a+}(L_1)$, then the semigroup property (6) is not fulfilled and is replaced [9] by

$$I^{\alpha}_{a+} I^{\beta}_{a+} f(x) = I^{\alpha+\beta}_{a+} f(x) - \sum_{k=0}^{n-1} \frac{A_k}{\Gamma(\alpha-k)} (x-a)^{\alpha-k-1};\ A_k = \partial_x^{n-k-1}\left(I^{n+\beta}_{a+} f\right)\Big|_{x=a}, \tag{8}$$

where $n = 1 - [\beta]$, $\alpha > 0$, $\beta > 0$; $[\beta]$ is integer part of fractional indices of order β; coefficients A_k are calculated at point $x = a+$. Further we shall use the composition formulae of fractional derivatives and the integral operators of type (8) for the case, when $0 < \alpha < 1$, $0 < \beta < 1$. These formulae can be obtained with the usage of the fractional derivative definition $D^{\alpha}_{a+} f(x) = \partial_x I^{1-\alpha}_{a+} f(x)$, of the relation $I^{\alpha}_{a+} f(x) = D^{-\alpha}_{a+} f(x)$ at $\alpha < 0$ and of the expression (8). As a result, we get

$$D^{\alpha}_{a+} I^{\beta}_{a+} f(x) = I^{\beta-\alpha}_{a+} f(x); \quad I^{\beta}_{a+} D^{\alpha}_{a+} f(x) = I^{\beta-\alpha}_{a+} f(x) - \varphi_1(x);$$

$$\left[D^{\alpha}_{a+}, I^{\beta}_{a+}\right] f(x) = \varphi_1(x); \quad \varphi_1 = b_1 (x-a)^{\beta-1}/\Gamma(\beta); \quad b_1 = I^{1-\alpha}_{a+} f(a). \tag{9}$$

At $\alpha = \beta$ from (9) follows the known composition formulae [9]

$$D_{a+}^{\alpha} I_{a+}^{\alpha} f(x) = f(x); \quad I_{a+}^{\alpha} D_{a+}^{\alpha} f(x) = f(x) - \varphi_2(x); \quad \varphi_2 = b_1 (x-a)^{\alpha-1} / \Gamma(\alpha). \tag{10}$$

For compositions and commutator of the fractional derivatives operators we find

$$D_{a+}^{\alpha} D_{a+}^{\beta} f = D_{a+}^{\alpha+\beta} f - \varphi_3; \quad D_{a+}^{\beta} D_{a+}^{\alpha} f = D_{a+}^{\alpha+\beta} f - \varphi_4; \quad \left[D_{a+}^{\alpha}, D_{a+}^{\beta}\right] f = \varphi_4 - \varphi_3;$$

$$\varphi_3 = b_2 / (x-a)^{1-\alpha}\ \Gamma(-\alpha); \quad \varphi_4 = b_1 / (x-a)^{1+\beta} \Gamma(-\beta); \quad b_2 = I_{a+}^{1-\beta} f(a). \tag{11}$$

If $b_1 = b_2 = 0$, then $\varphi_3 = \varphi_4 = 0$ and from (11) follows the semigroup property of type (6) and the fractional derivatives operators with different or equal indices of order are commuted

$$D_{a+}^{\alpha} D_{a+}^{\beta} f = D_{a+}^{\beta} D_{a+}^{\alpha} f; \quad \left[D_{a+}^{\alpha}, D_{a+}^{\beta}\right] f = 0. \tag{12}$$

If $\alpha = \beta$, then $b_1 = b_2$, $\varphi_3 = \varphi_4$ and at $b_2 \neq 0$ from (11) follows that the fractional derivatives operators are commuted, but the semigroup property of type (12) is not fulfilled and is replaced by (11). This makes it possible to conclude: if for the fractional derivatives operators the semigroup property (12) takes place, then they are commuted; the inverse confirmation is incorrect.

4. The abnormal behavior of the plastic subsystems of fractal string

First we consider only the plastic subsystems of fractal string, when all components of tensor a_{kn}, b_{bn} are zero except a_{11}, b_{11}. Then from (5) follows the equation

$$D_1(a_{11} D_1) u = F_1(b_{11} F_1) u; \; a_{11} = \rho_{11} c_1^2; \; b_{11} = \kappa_{11} g_1^2, \tag{13}$$

where ρ_{11}, κ_{11} are density and force parameter of plastic subsystems of fractal string. In adiabatic approximation, when ρ_{11} and κ_{11} follow the changed u without delay, parameters a_{11}, b_{11} can be considered as constants. Then equation (13) is simplified

$$D_t^{\nu} D_t^{\nu} u = v_1^2 D_x^{\alpha} D_x^{\alpha} u; \quad v_1^2 = b_{11} / a_{11}. \tag{14}$$

From (14) it follows that displacement u of a string must show fractal properties as in the process of time $(\nu \neq 0)$, so of space $(\alpha \neq 0)$ evolution. It should be noticed that in [7] physical interpretation as a part of system states, preserving for the time of evolution t' is given for the fractional index ν. In contrast to work [7] our model also takes into account the space fractal properties $(\alpha \neq 0)$ of a string. By analogy with [7] fractional index α can be interpreted as a part of system states preserving under the space evolution along the length x'. The solution of equation (14) can be found with the help of the method of separation of variables, that is to presuppose $u(x,t) = u_1(t)\, u_2(x)$. Then from (14) we receive two ordinary differential equations of fractional order for unknown functions u_1, u_2

$$D_t^{\nu} D_t^{\nu} u_1(t) = \lambda_1 u_1(t); \quad D_x^{\alpha} D_x^{\alpha} u_2(x) = \lambda_2 u_2(x); \quad \lambda_1 = \lambda_2\ v_1^2, \tag{15}$$

where λ_1, λ_2 are constants. Using the composition formulae (11) for the fractional derivatives operators from (15) we receive two inhomogeneous ordinary differential equations of fractional order

$$D_t^{2\nu} u_1 - \lambda_1 u_1 = \psi_1 ;\ D_x^{2\alpha} u_2 - \lambda_2 u_2 = \psi_2 ;\quad b_{\nu 1} = I_t^{1-\nu} u_1(t');$$

$$\psi_1 = b_{\nu 1} \left|t-t'\right|^{-1-\nu} / \Gamma(-\nu);\ \psi_2 = b_{\alpha 1} \left|x-x'\right|^{-1-\alpha} / \Gamma(-\alpha);\ b_{\alpha 1} = I_x^{1-\alpha} u_2(x'). \tag{16}$$

On the other hand, using the composition formulae (11) at once for equation (14) we obtain the inhomogeneos equation with the fractional partial derivatives

$$D_t^{2\nu} u(x,t) - \nu_1^2 D_x^{2\alpha} u(x,t) = \psi_3(x,t) - \psi_4(x,t);\ b_\nu(x) = I_t^{1-\nu} u(t',x);$$

$$\psi_3 = b_\nu(x)\left|t-t'\right|^{-1-\nu} / \Gamma(-\nu);\ \psi_4 = b_\alpha(t)\left|x-x'\right|^{-1-\alpha} / \Gamma(-\alpha);\ b_\alpha(t) = I_x^{1-\alpha} u(t,x') . \tag{17}$$

The received equations (13)–(17) are the basis for the investigation of the abnormal behavior of the plastic subsystems of fractal string.

In this paper we consider only solutions of the Cauchy task for equations (16). Let's notice that spectral task about the search of proper significance's λ_2 , λ_1 requires a special individual consideration. The solution of first equation from (16) with the initial condition $I_t^{1-2\nu} u_1(t') = b_1$ at $0 \le 2\nu \le 1$ is

$$u_1(t) = b_1 N_1(t) + N_2(t);\ N_2 = \int_{t'}^{t} \left|t-\tau\right|^{2\nu-1} E_{2\nu,2\nu}(z_2)\psi_1(\tau)d\tau;$$

$$N_1 = \left|t-t'\right|^{2\nu-1} E_{2\nu,2\nu}(z_1);\quad z_1 = \lambda_1 \left|t-t'\right|^{2\nu};\quad z_2 = \lambda_1 \left|t-\tau\right|^{2\nu}, \tag{18}$$

where Mittag–Leffler function $E_{\alpha,\beta}(z)$ is expressed [9] by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} z^k / \Gamma(\alpha k + \beta). \tag{19}$$

If $1 \le 2\nu \le 2$, then the solution of first equation from (16) with the initial conditions $D_t^{2\nu-1} u_1(t') = b_2$, $I_t^{2-2\nu} u_1(t') = b_3$ is

$$u_1(t) = b_2 N_1(t) + b_3 N_3(t) + N_2(t);\quad N_3 = \left|t-t'\right|^{2\nu-2} E_{2\nu,2\nu-1}(z_1). \tag{20}$$

The solutions (18), (20) have been obtained by analogy with [9]. First we act to the left on differential equation with the help of operator $I_t^{2\nu}$. Later the received integral equation is solved with the help of the method of the consistent approach. The structure of the solutions of the second equation from (16) coincides with (18) , (20). Therefore at $0 \le 2\alpha \le 1$ with the initial condition $I_x^{1-2\alpha} u_2(x') = S_1$ we find

$$u_2(x) = s_1 R_1(x) + R_2(x);\quad R_2 = \int_{x'}^{x} \left|x-\xi\right|^{2\alpha-1} E_{2\alpha,2\alpha}(z_4)\psi_2(\xi)d\xi$$

$$R_1 = \left|x-x'\right|^{2\alpha-1} E_{2\alpha,2\alpha}(z_3);\quad z_3 = \lambda_2 \left|x-x'\right|^{2\alpha};\quad z_4 = \lambda_2 \left|x-\xi\right|^{2\alpha}; \tag{21}$$

By analogy with (20), at $1 \le 2\alpha \le 2$ with the initial conditions $D_x^{2\alpha-1} u_2(x') = S_2$, $I_x^{2-2\alpha} u_2(x') = S_3$ we obtain

$$u_2(x) = s_2 R_1(x) + s_3 R_3(x) + R_2(x); \quad R_3 = x - x'^{\,2\alpha-2} E_{2\alpha,2\alpha-1}(z_3). \tag{22}$$

Then product of the found solutions $u_1(t)\, u_2(x)$ will be the solution of equation (14). In this case four variants of the solutions can be realized

$$u(x,t) = \begin{cases} (b_1 N_1 + N_2)(S_1 R_1 + R_2); & 0 \le 2\nu \le 1; \quad 0 \le 2\alpha \le 1; \\ (b_1 N_1 + N_2)(S_2 R_1 + S_3 R_3 + R_2); & 0 \le 2\nu \le 1; \quad 1 \le 2\alpha \le 2; \\ (b_2 N_1 + b_3 N_3 + N_2)(S_1 R_1 + R_2); & 1 \le 2\nu \le 2; \quad 0 \le 2\alpha \le 1; \\ (b_2 N_1 + b_3 N_3 + N_2)(S_2 R_1 + S_3 R_3 + R_2); & 1 \le 2\nu \le 2; \quad 1 \le 2\alpha \le 2. \end{cases} \tag{23}$$

The analysis of the received solutions (23) shows that by using equations (13)–(17) it is possible to describe principally new physical effects typical only of the plastic subsystems of the fractal string.

5. Discussion

At first we show that solutions (23) contain the solutions of ordinary equations as the limiting case. Under $\nu = 1$, $\alpha = 1$ from (16), (17) it follows that functions $\psi_1 = \psi_2 = \psi_3 = \psi_4 = 0$. Then from (17) an equation of the hyperbolic type is obtained (a wave equation)

$$\partial_t^2 u(x,t) = v_1^2\, \partial_x^2 u(x,t), \tag{24}$$

which describes the behavior of an ordinary string. The system (16) is written as

$$d_t^2 u_1(t) - \lambda_1 u_1(t) = 0; \quad d_x^2 u_2(x) - \lambda_2 u_2(x) = 0, \tag{25}$$

where $d_t = d/dt$, $d_x = d/dx$ are operators ordinary derivatives according to variables t, x. For limiting case function (19) is expressed through elementaries $E_{2,2}(z) = sh\, z_0 / z_0$, $E_{2,1}(z) = ch\, z_0$, $z_0 = z^{1/2}$. Therefore, using the expressions (18)-(22) under $\nu = 1$, $\alpha = 1$ we find

$$\begin{aligned} &N_1(t) = T_1\, sh\, a_1; \quad N_3(t) = ch\, a_1; \quad N_2 = 0; \quad a_1 = t - t'\, T_1^{-1}; \quad T_1 = \lambda_1^{-1/2}; \\ &R_1(x) = l_1\, sh\, a_2; \quad R_3(x) = ch\, a_2; \quad R_2 = 0; \quad a_2 = x - x'\, l_1^{-1}; \quad l_1 = \lambda_2^{-1/2}; \\ &b_2 = d_t u_1(t'); \quad b_3 = u_1(t'); \quad S_2 = d_x u_2(x'); \quad S_3 = u_2(x'). \end{aligned} \tag{26}$$

Substituting (26) into (23) or (18) , (21) , we receive the solutions of Cauchy task for equations (24) , (25). Using (15) , (26) under $\lambda_1 > 0$, $\lambda_2 > 0$ we find $l_1 = v_1 T_1$, where the parameter l_1, v_1, T_1 mean characteristic limiting length, speed, relaxation time in the plastic subsystems of the fractal string. If $\lambda_1 < 0$, $\lambda_2 < 0$, then from (26) follows

$$
\begin{aligned}
&N_1(t)=\omega_1^{-1}\sin a_3; && N_3(t)=\cos a_3; && a_3=\omega_1|t-t'|; && \omega_1=(-\lambda_1)^{1/2};\\
&R_1(x)=q_1^{-1}\sin a_4; && R_3(x)=\cos a_4; && a_4=q_1|x-x'|; && q_1=(-\lambda_2)^{1/2},
\end{aligned}
\tag{27}
$$

where $\omega_1 = q_1 v_1$ and the parameter q_1, ω_1 mean characteristic limiting wave number, oscillation frequency in the plastic subsystems of the fractal string.

Now, we consider the partial solutions from (23) for the next points (ν,α), where $\nu,\alpha = 1, 1/2, 0$. Using the expressions (18)–(22) under $\nu = 1/2$, $\alpha = 1/2$ and formulae $E_{1,1}(z) = \exp z$, $E_{1,0}(z) = z E_{1,1}(z) = z\exp z$ we obtain

$$
\begin{aligned}
&N_1(t)=\exp(\lambda_1|t-t'|); && R_1(x)=\exp(\lambda_2|x-x'|); && N_3=\lambda_1 N_1; && R_3=\lambda_2 R_1;\\
&b_1=b_2=u_1(t'); && b_3=0; && S_1=S_2=u_2(x'); && S_3=0.
\end{aligned}
\tag{28}
$$

As against (26) now the functions N_2, R_2 are not zero

$$
N_2(t)=\int_{t'}^{t}\psi_1(\tau)\exp(\lambda_1|t-\tau|)d\tau; \quad R_2(x)=\int_{x'}^{x}\psi_2(\xi)\exp(\lambda_2|x-\xi|)d\xi. \tag{29}
$$

They determine the contribution in solutions (23) from right parts of equations (16). For the points $(\nu,\alpha)=(1/2,1)$ or $(1,1/2)$ from (17) follow the equations of the parabolic type with the partial derivatives of the integer order

$$
\begin{aligned}
&\partial_t u(x,t)-v_1^2\,\partial_x^2 u(x,t)=-r_1(x)|t-t'|^{-3/2}; && r_1=b_\nu(x)/2\pi^{1/2},\\
&\partial_t^2 u(x,t)-v_1^2\,\partial_x u(x,t)=r_2(t)|x-x'|^{-3/2}; && r_2=b_\alpha(t)/2\pi^{1/2}.
\end{aligned}
\tag{30}
$$

For the point (1/2, 1/2) we obtain an equation containing only the first-order partial derivatives

$$
\partial_t u(x,t)-v_1^2\,\partial_x u(x,t)=r_2(t)|x-x'|^{-3/2}-r_1(x)|t-t'|^{-3/2}. \tag{31}
$$

Let's notice, that the right parts of equations (16) , (30) ,(31) appear, if our model takes into account time $(\nu\neq 0)$ or space $(\alpha\neq 0)$ fractal properties of a string. The analysis of the received expressions (24)–(31) shows that the new physical effects can be observed as the oscillational diffusion or the diffusional oscillations.

Let's notice, that the real structure of the lattice of magnets, ferroelectrics, ferroelastics, high-temperature superconductors is typical of the presence of areas of quasi-one-dimensional chains of atoms having finite length with local spontaneous deformation in the ground state (microstrings) or of quasi-two-dimensional formations of cluster type [3]. On the other hand, it is known [10], that near a series of phase transitions (structural, ferroelectric, superconducting ones) in narrow temperature intervals there occur the spatially inhomogeneous states in the form of irregular or regular structures. The proposed in [10] theoretical model of the initiation and evolution of such structures is based on the striction mechanism (the connection of elastic deformation with the spontaneous polarization) and on representations about the spatial fluctuations of the phase transition temperature. The numerical simulation method has been used in [11] to investigate the behavior of a quasi-one-dimensional nonlinear lattice with the spontaneous deformation, also the mechanism of the regular-to irregular mode transition through different bifurcation sequences have been found (with the example of the oscillation processes with the attenuation and purely relaxation processes). We'd like to remind , that distances from exponential to the side of power dependence of the relaxational process parameters were experimentally established for some fractal physical objects [1]. Theoretical approach with the usage of equations with fractional time derivatives was successfully approved for the explanation of delayed relaxation processes of polarization of dielectrics in work [7]. The analysis of the received partial solutions of the equation (13)–(17) shows that our model of the fractal string can be used to investigate the fractal and stochastic properties of the nonlinear lattice with the spontaneous deformation in the vicinity of different phase transitions.

6. Conclusion

In this paper the basic equations with fractional space-time derivatives for the description of the fractal string model are introduced. The obtained results under the research of Cauchy task for the plastic subsystems of fractal string can be used under the solution of different boundary tasks. The model of the fractal string can be used to explain the anomalies of a number of physical parameters in the region of different equilibrium and nonequilibrium phase transitions.

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