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## YIELD STRENGTH OF A MATERIAL PRE-PROCESSED BY SIMPLE SHEAR

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*Modern techniques of severe plastic deformation used as a means for grain refinement in metallic materials rely on simple shear as the main deformation mode. Prediction of the mechanical properties of the processed materials under tensile loading is a formidable task as commonly no universal, strain path independent constitutive laws hold. In this paper we derive an analytical relation that makes it possible to predict the mechanical response to uniaxial tensile loading for a material that has been pre-processed by simple shear and presents a linear strain gradient in it. A facile recipe for mechanical tests on solid bars required for this prediction to be made is proposed. As a trial, it has been exercised for the case of commercial purity copper rods. The results of the derivation of the true stress-strain curve for large tensile deformation of copper are presented. The method proposed is recommended for design with metallic materials that underwent pre-processing by simple shear.*

**Keywords:** strain hardening, torsion, tension, strain gradient, strain path change, copper

*Для прогнозу властивостей субмікросталічних металів, отриманих методами інтенсивних пластичних деформацій, необхідно знати напруження при одновісному пластичному розтягуванні після обробки великим простим зсувом (деформація зсуву більше 10). Механічні властивості матеріалів при таких непропорційних шляхах деформування вивчають в основному за допомогою трубчастих*

зразків. Через втрату стійкості трубок при крученні такі експерименти можливі лише для малих пружньопластичних деформацій, що не перевищують кількох відсотків. У статті запропоновано й обґрунтовано метод визначення напруги течії матеріалу, попередньо обробленого великим простим зсувом. Метод заснований на двох стандартних випробуваннях: крученні з вільними торцями й одновісному розтягуванні. Отримано співвідношення, що дозволяє по нарузі течії неоднорідного зразка, попередньо підданого крученню, знайти напругу пластичного розтягування його поверхневого шару з певною деформацією простого зсуву. Шляхом простих аналітичних оцінок показано, що пружні залишкові напруги першого роду, які виникають після розвантаження зразка, попередньо підданого крученню, практично не впливають на межу тікучості при розтягуванні.

**Ключові слова:** деформаційне зміцнення, кручення, розтягування, градієнт деформації, зміна шляху деформування, мідь

## 1. Introduction

Processing of metals by simple shear is a good way to improve their mechanical characteristics. This comes to bearing especially in the production of ultrafine grained materials by severe plastic deformation (SPD). SPD techniques, such as high-pressure torsion, high-pressure tube twisting, equal-channel angular pressing, twist extrusion, and shear extrusion have one thing in common – they all are based on simple shear [1,2] and almost all of them present a strain gradient in them. Products fabricated by these techniques are often designed for structures that operate under extremely large tensile loads. Design with such materials therefore requires a reliable tool for predicting the flow stress for metals, which underwent processing by gradient simple shear of a given magnitude.

The large strain behavior of metals is usually studied in torsion of cylindrical bars because very large strains can be readily achieved in torsion. Indeed, during tensile testing – which is the most commonly employed characterization technique – the uniform deformation is limited because of early necking. However, the mechanism of strain hardening is quite special for torsion because of the small number of the operating slip systems that lead to smaller equivalent stresses for torsion compared to tension or compression [3,4]. This presents difficulties in the characterization of the material behavior at large strains.

The problem of construction of stress-strain curves for torsion of solid bars has been resolved by Fields and Backofen [5] who established a formula for obtaining the flow stress at the outer radius of the twisted bar. However, no such formula is available for tension of a sample with a strain gradient, particularly when a bar is tested in tension after being pre-twisted in torsion. This problem is resolved in the present paper. The importance of such testing is that stress-strain curves can be obtained for tension for very large strains, up to the same strains as in torsion, by tension of pre-twisted bars. For this purpose only thin-walled tubes were pre-twisted so far where the strain gradient can be neglected [6–12]. However, the maximum plastic strain is very limited in torsion of such tubes, which is not the case for the torsion of solid bars.

In the following we first present the theoretical basis and then show that the role of the residual stresses, which are inherent in gradient structures, can be ne-

glected at large strains. Finally, the new technique is applied for the large strain torsion of copper bars.

## 2. Theoretical basis

We consider a solid cylindrical bar twisted in large strain torsion. It can be shown with using the equilibrium equation that for large uniform torsion, the local plastic shear strain  $\gamma_r$  is proportional to the local radius  $r$  [13]:

$$\gamma_r = \frac{r}{R} \gamma_R. \quad (1)$$

Here  $\gamma_R$  is the shear strain at the outer radius  $R$  of the sample. When the twisted bar is subjected to tensile testing, the local tensile flow stress depends on the tensile true strain  $\varepsilon$  and on the local shear pre-strain  $\gamma_r$ ;  $\sigma(r) = \sigma(\varepsilon, \gamma_r)$ . The force required for plastic stretching of a rod previously subjected by torsion is given by

$$F(\varepsilon, \gamma_R) = 2\pi \int_0^R \sigma(\varepsilon, \gamma_r) r dr. \quad (2)$$

This integral can be developed as follows:

$$F(\varepsilon, \gamma_R) = 2\pi \int_0^R \sigma\left(\varepsilon, \frac{r\gamma_R}{R}\right) r dr = \frac{2S}{\gamma_R^2} \int_0^{\gamma_R} \sigma(\varepsilon, \gamma_r) \gamma_r d\gamma_r, \quad (3)$$

where  $S$  is the cross-sectional area of the bar. We introduce the quantity  $\bar{\sigma}(\varepsilon, \gamma_R)$ , which is the apparent tensile flow stress of the bar:

$$\bar{\sigma}(\varepsilon, \gamma_R) = \frac{F(\varepsilon, \gamma_R)}{S} = \frac{2}{\gamma_R^2} \int_0^{\gamma_R} \sigma(\varepsilon, \gamma_r) \gamma_r d\gamma_r. \quad (4)$$

After differentiation with respect to  $\gamma_R$ , the following expression is obtained:

$$\frac{\partial \bar{\sigma}(\varepsilon, \gamma_R)}{\partial \gamma_R} = \frac{2}{\gamma_R} [\sigma(\varepsilon, \gamma_R) - \bar{\sigma}(\varepsilon, \gamma_R)]. \quad (5)$$

Hence, it follows:

$$\sigma(\varepsilon, \gamma_R) = \bar{\sigma}(\varepsilon, \gamma_R) + \frac{\gamma_R}{2} \frac{\partial \bar{\sigma}(\varepsilon, \gamma_R)}{\partial \gamma_R}. \quad (6)$$

This formula allows one to find the stress-strain curve  $\sigma = \sigma(\varepsilon, \gamma_R)$  using the experimentally measured curve for the apparent stress  $\bar{\sigma}(\varepsilon, \gamma_R)$  and its derivative with respect to  $\gamma_R$ .

## 3. The role of residual stresses

Residual stresses arise after unloading a plastically twisted solid bar sample [13]. Assuming that the entire cross section of the sample was in plastic state un-

der torsion, the local residual shear stress  $\tau_{\text{res}}(r, \gamma_R)$  after a shear strain  $\gamma_R$  reached at the outer radius of the bar is given by the following equation [14]:

$$\tau_{\text{res}}(r, \gamma_R) = \tau(r, \gamma_R) - \frac{4}{3} \frac{r}{R} \bar{\tau}(\gamma_R). \quad (7)$$

Here  $\tau(r, \gamma_R)$  is the local shear flow stress before unloading and  $\bar{\tau}$  is the mean shear stress across the bar,

$$\bar{\tau}(\gamma_R) = \frac{3}{R^3} \int_0^R \tau(r, \gamma_R) r^2 dr. \quad (8)$$

The effect of the residual stresses during the subsequent tensile testing is a reduction of the yield stress of the rod in tension, as they are present in the yield condition. It can be shown, however, that the residual stresses can be ignored when the technique presented above is applied. This is due to a rapid relaxation of the residual stresses at the beginning of plastic deformation. As will be shown below, the strain required for this relaxation is very small. The residual elastic strain  $\gamma_{\text{res}}$  associated to the residual elastic stress  $\tau_{\text{res}}$  is

$$\gamma_{\text{res}} = \frac{\tau_{\text{res}}}{G}, \quad (9)$$

where  $G$  is the elastic shear modulus. According to the associated flow rule, the following relation between the components of the strain increments and the acting stresses is valid:

$$d\gamma = \frac{3\tau_{\text{res}}}{\sigma} de_{VM}, \quad (10)$$

where  $e_{VM}$  is the von Mises equivalent strain. It follows then from Eqs. (9) and (10) that

$$d\gamma_{\text{res}} = -\gamma_{\text{res}} \frac{3G}{\sigma} de_{VM}. \quad (11)$$

Here the negative sign takes into account that  $\gamma_{\text{res}}$  decreases in absolute value, so that the sign of  $d\gamma_{\text{res}}$  is opposite to the sign of  $\gamma_{\text{res}}$ . By integrating Eq. (11) at a constant stress  $\sigma$ , we obtain

$$\gamma_{\text{res}} \sim \exp\left(-\frac{3G}{\sigma} e_{VM}\right). \quad (12)$$

Therefore, the characteristic equivalent plastic strain  $e_{VM}$  required for the relaxation of residual stresses for torsion-tension can be estimated as

$$e_{VM} = \frac{\sigma}{3G}. \quad (13)$$

Using characteristic stress values that correspond to the tensile test of copper after torsion, the following estimate is obtained:  $e_{VM} \sim 10^{-3}$ . This estimate shows that already after a very small tensile strain, the effect of the residual stresses can be

neglected. The physical meaning of the above analysis is that the stress state moves along the yield surface very rapidly from a combined tension-torsion into a pure tension state during the initial stage of the tensile test.

#### 4. Experimental results

Experiments were carried out on commercially pure copper samples at room temperature. The initial microstructure of the material can be characterized by an average grain size of about 30  $\mu\text{m}$  with a weak crystallographic texture. The dimensions of the deforming part of the samples were as follows: 7 mm in diameter and 40 mm in gauge length. The torsion testing was done in a free-end torsion machine to different rotation angles at a constant angular speed of 0.2 rad/s. The selected values of the rotation angle (in radian) were: 11.43, 22.85, 34.29, 45.71, 57.14, and 68.57. They correspond to a shear strain of 1, 2, 3, 4, 5, and 6, at the outer radius of the sample, respectively. These values were converted into the von Mises equivalent strain using the formula  $e_{VM} = \gamma/\sqrt{3}$ . The small lengthening of the bar during the free end torsion testing (less than 2%) was neglected in the analysis of the experimental data. The shear flow stress acting at the outer radius of the bar was calculated by the Fields and Backofen formula [5] and converted into equivalent von Mises stress using the formula  $\sigma = \sqrt{3}\tau$ . The obtained stress-strain curve for torsion curve is displayed in Fig. 1 (curve 2).

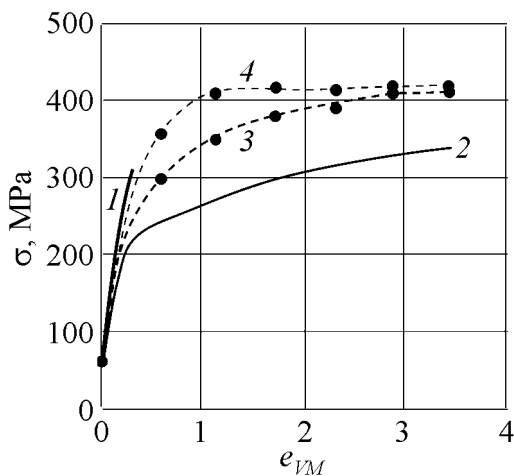
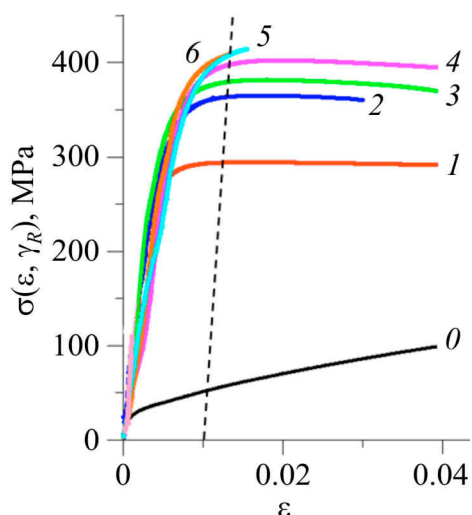


Fig. 1. Stress-strain curves obtained for pure copper solid bars in tension (curve 1), in torsion (curve 2) and in tension after torsion with different magnitude of the twist (curve 3). Curve 4 was constructed using Eq. (6)

The tensile tests were done in a 10 ton Zwick machine at a strain rate of 0.05 mm/s. The results are shown in Fig. 2 for the twisted samples and in Fig. 1 for the non-twisted ones for larger strain (curve 1). After torsion rupture took place under tension already after about 2–4% plastic strain. With the available specimens, six points on the average stress-strain curve were obtained (labeled 3 in Fig. 1). They were taken from the tensile curves at 1% strain. A continuous curve was fitted to these points to calculate the derivative in Eq. (6), and the resulting large strain tensile test curve based on Eq. (6) was plotted as curve 4. It can be seen that the initial part of this curve matches the continuous tensile curve well. At large deformations, from about a strain of 1.5, the curve levels off at a constant stress level of about 415 MPa.



**Fig. 2.** Stress-strain curves in tension obtained without pre-torsion (0) and after different amounts of shear in torsion  $\gamma$ : 1 – 1, 2 – 2, 3 – 3, 4 – 4, 5 – 5, 6 – 6

The stress-strain curve 4 in Fig. 1 obtained for tensile testing after torsion was constructed for the material behavior at the outer radius of the twisted bar, but it is also valid for the inner points in the bulk of the bar.

## 5. Discussion

As can be seen from Fig. 1, the monotonic torsion and tensile stress-strain curves do not coincide, despite the use of the equivalent stress and strain quantities as a common platform: the torsion curve lies below the tensile one. This effect known for a long time and was examined in the past [3,4]. The main reason for it was mentioned in Section 1, *viz.* the scarcity of slip systems in torsion compared to tension.

One particularity of the present results is that for low tensile strains the tensile flow stress after torsion agrees well with the flow stress in monotonic tension. Indeed, lower stresses are expected for a strain path change because the microstructure that develops in the first path is not stable for the new path, thus many dislocations that were immobile in the first path can glide in the second one. However, it has been shown in Ref. [4] that the dislocation density is higher in torsion compared to tension, which can compensate for this effect.

Another particularity of the results in Fig. 1 is that the tensile flow stress is constant after torsion at large strains, starting from about 1.5 strain. This effect was observed for the first time because the present technique is the first one to provide access to tensile flow stress after large strain torsion. Its origin might be rooted in the fragmentation of the grains which occurs under severe plastic deformation [3,15]. Further studies are needed to identify the exact reasons for this material behavior.

An interesting observation can be drawn from the stress-strain curves in Fig. 1, namely, it is apparent that the equivalent flow stress is higher in tension than in torsion. This effect is due to the anisotropy that develops within the bar; a non-isotropic orientation distribution of grain orientations, *i.e.* a shear texture, appears [3,16]. Therefore, the tensile yield strength can be significantly enhanced through

pre-processing the material by simple shear. It then follows that less energy is required for strengthening the material by simple shear compared to tensile straining.

## 6. Conclusions

In this work we presented a new theoretical derivation for obtaining tensile stress-strain curves for the bars pre-strained by torsion. Experiments on Cu providing exemplary data for the proposed derivation were conducted. The following main conclusions can be drawn:

1. In spite of the strain gradient inherent in a torsion-deformed bar, it is possible to obtain tensile stress-strain curves for it. The procedure includes a series of torsion tests of bars to different amounts of strains, followed by deforming them in uniaxial extension.

2. Application of the proposed algorithm to large strain torsion of Cu yielded a tensile stress-strain curve which saturates quite early, from an equivalent von Mises strain of about 1.

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## ПРЕДЕЛ ПРОЧНОСТИ МАТЕРИАЛОВ, ОБРАБОТАННЫХ ПРИ ПОМОЩИ ПРОСТОГО СДВИГА

Для прогноза свойств субмикроструктурных металлов, полученных методами интенсивных пластических деформаций, необходимо знать напряжение их течения при одноосном растяжении после обработки простым сдвигом большой величины (деформация сдвига более 10). Механические свойства материалов при таких непропорциональных путях деформирования изучают в основном с помощью трубчатых образцов. Из-за потери устойчивости трубок при кручении такие эксперименты возможны лишь для малых упругопластических деформаций, не превышающих нескольких процентов. В статье предложен и обоснован метод определения напряжения течения материала, предварительно обработанного большим простым сдвигом. Метод основан на двух стандартных испытаниях: кручении со свободными торцами и одноосном растяжении. Получено соотношение, позволяющее по напряжению течения неоднородного образца, предварительно подвергнутого кручению, найти напряжение пластического растяжения его поверхностного слоя с определенной деформацией простого сдвига. Путем простых аналитических оценок показано, что упругие остаточные напряжения первого рода, возникающие после разгрузки образца, предварительно подвергнутого кручению, практически не влияют на предел текучести при растяжении.

**Ключевые слова:** деформационное упрочнение, кручение, растяжение, градиент деформации, изменение пути деформирования, медь